GROUPS OF ORDER 48

SUMMARY

order	48
# abelian	5
# other decomposable	24
# other indecomposable	23
TOTAL	52

48 = $\langle A^{48} \rangle$ = 16 × 3

48.02 = 24 × 2 = $\langle A^{24}, B^2 \rangle$

48.03 = 12 × 4 = $\langle A^{12}, B^4 \rangle$

48.04 = 12 × 2^2 = $\langle A^{12}, B^2, C^2 \rangle$

48.05 = 6 × 2^3 = $\langle A^6, B^2, C^2, D^2 \rangle$

48.06 = D_8 × 6 = $\langle A^4, B^2, C^6, BA = A^{-1}B \rangle$

48.07 = Q_8 × 6 = $\langle A^4, B^4 = A^2, C^6, BA = A^{-1}B \rangle$

48.08 = 16.08 × 3 = $\langle A^4, B^2, C^2, D^3, CB = A^2BC \rangle$

48.09 = 16.09 × 3 = $\langle A^4, B^2, C^2, D^3, CA = ABC \rangle$

48.10 = $D_{4,4}$ × 3 = $\langle A^4, B^4, C^3, BA = A^{-1}B \rangle$

48.11 = $M_{8,2}^{(5)}$ × 3 = $\langle A^8, B^2, C^3, BA = A^5B \rangle$

48.11 = $M_{8,2}^{(5)}$ × 3 = $\langle A^8, B^2, C^3, BA = A^5B \rangle$

48.12 = D_{16} × 3 = $\langle A^8, B^2, C^3, BA = A^{-1}B \rangle$

48.13 = $M_{8,2}^{(3)}$ × 3 = $\langle A^8, B^2, C^3, BA = A^3B \rangle$

48.14 = Q_{16} × 3 = $\langle A^8, B^2 = A^4, C^3, BA = A^{-1}B \rangle$

48.15 = $D_6 \times 2^3 = \langle A^3, B^2, C^2, D^2, E^2, BA = A^{-1}B \rangle$

48.16 = $D_6 \times 4 \times 2 = \langle A^3, B^2, C^4, D^2, BA = A^{-1}B \rangle$

48.17 = $D_{3,4} \times 2^2 = \langle A^3, B^4, C^2, D^2, BA = A^{-1}B \rangle$

48.18 = $D_{3,4} \times 4 = \langle A^3, B^4, C^4, BA = A^{-1}B \rangle$

48.19 = $D_6 \times 8 = \langle A^3, B^2, C^8, BA = A^{-1}B \rangle$

48.20 = $D_{3,8} \times 2 = \langle A^3, B^8, C^2, BA = A^{-1}B \rangle$

48.21 = $D_{3,16}$ = ⟨A^3, B^{16}, $BA = A^{-1}B$⟩

1	2	3	4	5	6	7	8	9	10	11	12
1	A A^2	B^8	AB^8 A^2B^8	B^4	AB^4 A^2B^4	B^{12}	AB^{12} A^2B^{12}	B^2	B^6	B^{10}	B^{14}

13	14	15	16	17	18	19	20	21	22	23	24
AB^2 A^2B^2	AB^6 A^2B^6	AB^{10} A^2B^{10}	AB^{14} A^2B^{14}	A^nB	A^nB^3	A^nB^5	A^nB^7	A^nB^9	A^nB^{11}	A^nB^{13}	A^nB^{15}

C	1	2	3	4	5	6	7	8	9	10	11	12	
#	1	2	1	2	1	2	1	2	1	1	1	3	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_3	1	1	1	1	1	1	1	1	–1	–1	–1	–1	2
χ_4	1	1	1	1	1	1	1	1	–1	–1	–1	–1	2
χ_5	1	1	1	1	–1	–1	–1	–1	i	–i	I	–i	3
χ_6	1	1	1	1	–1	–1	–1	–1	–i	i	–i	i	3
χ_7	1	1	1	1	–1	–1	–1	–1	i	–i	I	–i	3
χ_8	1	1	1	1	–1	–1	–1	–1	–i	i	–i	i	3
χ_9	1	1	–1	–1	i	i	–i	–i	θ^2	$i\theta^2$	$-\theta^2$	$-i\theta^2$	4
χ_{10}	1	1	–1	–1	–i	–i	i	i	$i\theta^2$	θ^2	$-i\theta^2$	$-\theta^2$	4
χ_{11}	1	1	–1	–1	i	i	–i	–i	$-\theta^2$	$-i\theta^2$	θ^2	$i\theta^2$	4
χ_{12}	1	1	–1	–1	–i	–i	i	i	$-i\theta^2$	$-\theta^2$	$i\theta^2$	θ^2	4
χ_{13}	1	1	–1	–1	i	i	–i	–i	θ^2	$i\theta^2$	$-\theta^2$	$-i\theta^2$	4
χ_{14}	1	1	–1	–1	–i	–i	i	i	$i\theta^2$	θ^2	$-i\theta^2$	$-\theta^2$	4
χ_{15}	1	1	–1	–1	i	i	–i	–i	$-\theta^2$	$-i\theta^2$	θ^2	$i\theta^2$	4
χ_{16}	1	1	–1	–1	–i	–i	i	i	$-i\theta^2$	$-\theta^2$	$i\theta^2$	θ^2	4
χ_{17}	2	–1	2	–1	2	–1	2	–1	2	2	2	2	5
χ_{18}	2	–1	2	–1	2	–1	2	–1	–2	–2	–2	–2	6
χ_{19}	2	–1	2	–1	–2	1	–2	1	2i	–2i	2i	–2i	7
χ_{20}	2	–1	2	–1	–2	1	–2	1	–2i	2i	–2i	2i	7
χ_{21}	2	–1	–2	1	2i	–i	–2i	i	$2\theta^2$	$2i\theta^2$	$-2\theta^2$	$-2i\theta^2$	0
χ_{22}	2	–1	–2	1	–2i	i	2i	–i	$-2i\theta^2$	$-2\theta^2$	$2i\theta^2$	$2\theta^2$	0
χ_{23}	2	–1	–2	1	2i	–i	–2i	i	$-2\theta^2$	$-2i\theta^2$	$2\theta^2$	$2i\theta^2$	0
χ_{24}	2	–1	–2	1	–2i	i	2i	–i	$2i\theta^2$	$2\theta^2$	$-2i\theta^2$	$-2\theta^2$	0
^	1	3	4	12	4	12	4	12	8	8	8	8	

C	13	14	15	16	17	18	19	20	21	22	23	24	
#	2	2	2	2	3	3	3	3	3	3	3	3	$\mathfrak{K}$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1
χ_3	−1	−1	−1	−1	i	−i	i	−i	i	−i	i	−i	2
χ_4	−1	−1	−1	−1	−i	I	−i	i	−i	I	−i	i	2
χ_5	i	−i	i	−i	θ^2	$i\theta^2$	$-\theta^2$	$-i\theta^2$	θ^2	$i\theta^2$	$-\theta^2$	$-i\theta^2$	3
χ_6	−i	i	−i	i	$i\theta^2$	θ^2	$-i\theta^2$	$-\theta^2$	$i\theta^2$	θ^2	$-\theta^3$	$-\theta^2$	3
χ_7	i	−i	i	−i	$-\theta^2$	$-i\theta^2$	θ^2	$i\theta^2$	$-\theta^2$	$-i\theta^2$	θ^2	$i\theta^2$	3
χ_8	−i	i	−i	−i	$i\theta^2$	$-\theta^2$	$i\theta^2$	θ^2	$-i\theta^2$	$-\theta^2$	θ^3	θ^2	3
χ_9	θ^2	$i\theta^2$	$-\theta^2$	$-i\theta^2$	θ	θ^3	$i\theta$	$i\theta^3$	$-\theta$	$-\theta^3$	$-i\theta$	$-i\theta^3$	4
χ_{10}	$i\theta^2$	θ^2	$-i\theta^2$	$-\theta^2$	θ^3	$-\theta$	$-i\theta^3$	$-\theta$	$-\theta^3$	θ	$i\theta^3$	$-\theta^5$	4
χ_{11}	$-\theta^2$	$-i\theta^2$	θ^2	$i\theta^2$	$i\theta$	$-i\theta^3$	$-\theta$	θ^3	$-i\theta$	$i\theta^3$	θ	$-\theta^3$	4
χ_{12}	$-i\theta^2$	$-\theta^2$	$i\theta^2$	θ^2	$i\theta^3$	$i\theta$	θ^3	$i\theta$	$-i\theta^3$	$-i\theta$	$-\theta^3$	$i\theta^5$	4
χ_{13}	θ^2	$i\theta^2$	$-\theta^2$	$-i\theta^2$	$-\theta$	$-\theta^3$	$-i\theta$	$-i\theta^3$	θ	θ^3	$i\theta$	$i\theta^3$	4
χ_{14}	$i\theta^2$	θ^2	$-i\theta^2$	$-\theta^2$	$-\theta^3$	θ	$i\theta^3$	θ	θ^3	$-\theta$	$-i\theta^3$	θ^5	4
χ_{15}	$-\theta^2$	$-i\theta^2$	θ^2	$i\theta^2$	$-i\theta$	$i\theta^3$	θ	$-\theta^3$	$i\theta$	$-i\theta^3$	$-\theta$	θ^3	4
χ_{16}	$-i\theta^2$	$-\theta^2$	$i\theta^2$	θ^2	$-i\theta^3$	$-i\theta$	$-\theta^3$	$-i\theta$	$i\theta^3$	$i\theta$	θ^3	$-i\theta^5$	4
χ_{17}	−1	−1	−1	−1	0	0	0	0	0	0	0	0	5
χ_{18}	1	1	1	1	0	0	0	0	0	0	0	0	6
χ_{19}	−i	i	−i	i	0	0	0	0	0	0	0	0	7
χ_{20}	i	−i	i	−i	0	0	0	0	0	0	0	0	7
χ_{21}	$-\theta^2$	$-i\theta^2$	θ^2	$i\theta^2$	0	0	0	0	0	0	0	0	0
χ_{22}	$i\theta^2$	θ^2	$-i\theta^2$	$-\theta^2$	0	0	0	0	0	0	0	0	0
χ_{23}	θ^2	$i\theta^2$	$-\theta^2$	$-i\theta^2$	0	0	0	0	0	0	0	0	0
χ_{24}	$-i\theta^2$	$-\theta^2$	$i\theta^2$	θ^2	0	0	0	0	0	0	0	0	0
^	24	24	24	24	16	16	16	16	16	16	16	16	

	Classes	H	G/H	∩	
1	1+2+3+4+5+6+7+8+9+10+11+12+13+14+15+16	**24**	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	**12**	**4**	2	
3	1 + 2 + 3 + 4	**6**	**8**	3	
4	1 + 2	**3**	**16**	4	**G′**
5	1 + 3 + 5 + 7 + 9 + 10 + 11 + 12	**8**	**D_6**	5	**Z = $\mathfrak{N}$ = Φ**
6	1 + 3 + 5 + 7	**4**	**12.04**	6	
7	1 + 3	**2**	**24.09**	7	

Sylow subgroups: [**16**] × 3, [**3**] **Maximal subgroups:** [**24**], [**16**] × 3

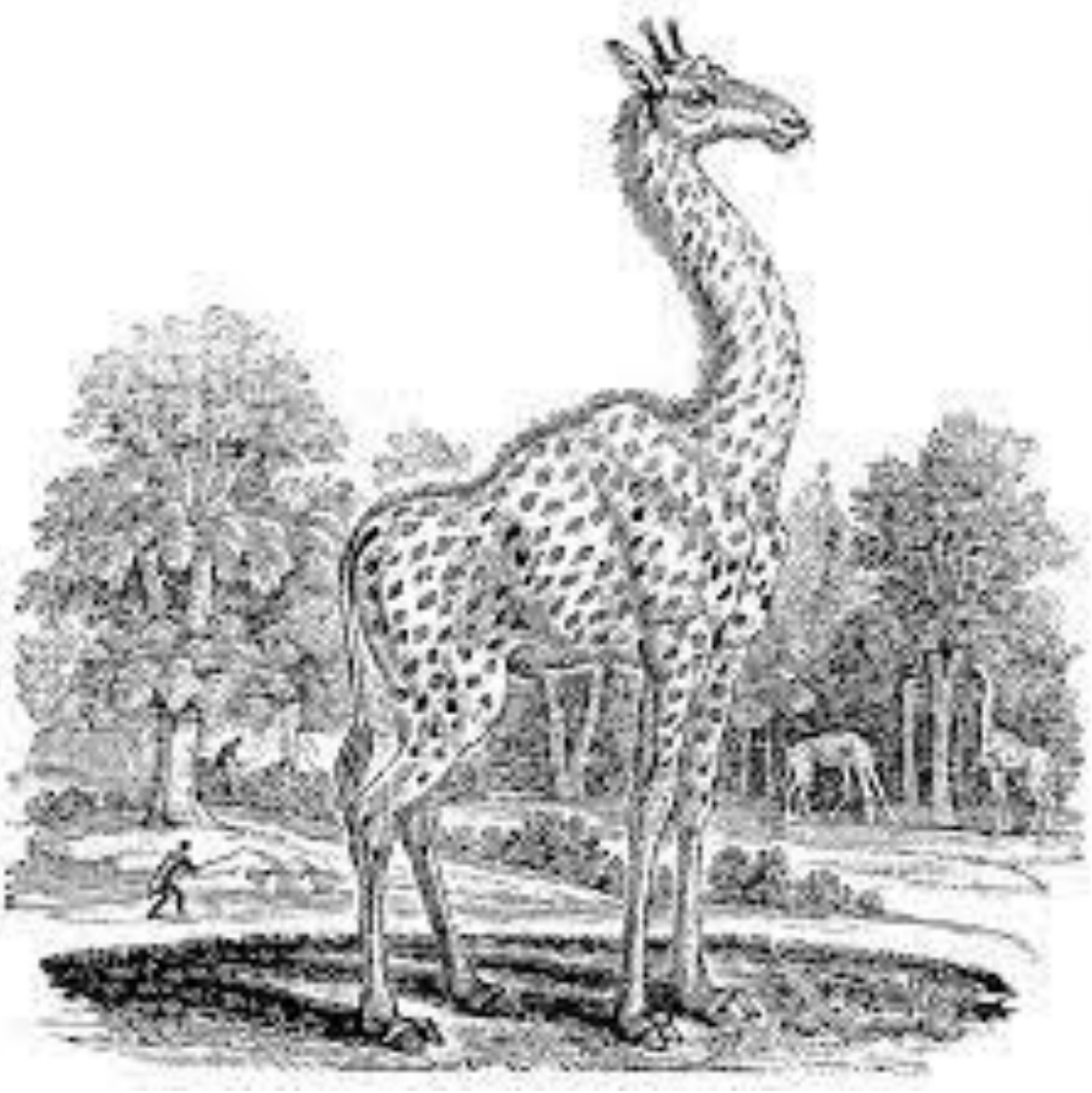
48.22 = $A_4 \times 2^2 = \langle A^2, B^2, C^3, D^2, E^2, CA = ABC, CB = AC \rangle$

48.23 = $A_4 \times 4 = \langle A^2, B^2, C^3, D^4, CA = ABC, CB = AC \rangle$

48.24 = 24.11 × 2 = $\langle A^3, B^4, C^2, D^2, BA = A^{-1}B, CB = B^{-1}C \rangle$

48.25 = $D_{24} \times 2 = \langle A^{12}, B^2, C^2, BA = A^{-1}B \rangle$

48.26 = $Q_{24} \times 2 = \langle A^{12}, B^2 = A^6, C^2, BA = A^{-1}B \rangle$



48.27 = ⟨A^2, B^3, C^2, D^4, DB = B^{-1}D, DC = ACD⟩

CC	1	2	3	4	5	6	7	8	9	10
Elts	1	A	D^2	AD^2	B B^2	AB AB^2	BD^2 B^2D^2	ABD^2 AB^2D^2	C AC	ACD CD^2

CC	11	12	13	14	15	16	17	18
elts	BC AB^2C	ABC B^2C	BCD^2 AB^2CD^2	B^2CD^2 $ABCD^2$	A^mB^nD	$A^mB^nD^3$	A^mB^nCD	$A^mB^nCD^3$

C	1	2	3	4	5	6	7	8	9	10	
#	1	1	1	1	2	2	2	2	2	2	ℋ
χ_1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	1
χ_3	1	1	1	1	1	1	1	1	–1	–1	2
χ_4	1	1	1	1	1	1	1	1	–1	–1	3
χ_5	1	1	–1	–1	1	1	–1	–1	1	–1	4
χ_6	1	1	–1	–1	1	1	–1	–1	1	–1	4
χ_7	1	1	–1	–1	1	1	–1	–1	–1	1	5
χ_8	1	1	–1	–1	1	1	–1	–1	–1	1	5
χ_9	2	2	2	2	–1	–1	–1	–1	2	2	6
χ_{10}	2	2	2	2	–1	–1	–1	–1	–2	–2	7
χ_{11}	2	2	–2	–2	–1	–1	1	1	2	–2	8
χ_{12}	2	2	–2	–2	–1	–1	1	1	–2	2	9
χ_{13}	2	–2	2	–2	2	–2	2	–2	0	0	10
χ_{14}	2	–2	–2	2	2	–2	–2	2	0	0	11
χ_{15}	2	–2	2	–2	–1	1	–1	1	0	0	12
χ_{16}	2	–2	2	–2	–1	1	–1	1	0	0	12
χ_{17}	2	–2	–2	2	–1	1	1	–1	0	0	13
χ_{18}	2	–2	–2	2	–1	1	1	–1	0	0	13
^	1	2	2	2	3	6	6	6	2	2	

C	11	12	13	14	15	16	17	18	
#	2	2	2	2	6	6	6	6	ℋ
χ_1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	–1	–1	–1	–1	1
χ_3	–1	–1	–1	–1	1	1	–1	–1	2
χ_4	–1	–1	–1	–1	–1	–1	1	1	3
χ_5	1	1	–1	–1	i	–i	i	–i	4
χ_6	1	1	–1	–1	–i	i	–i	i	4
χ_7	–1	–1	1	1	i	–i	–i	i	5
χ_8	–1	–1	1	1	–i	i	i	–i	5

C	11	12	13	14	15	16	17	18	
#	2	2	2	2	6	6	6	6	$\mathfrak{K}$
χ_9	−1	−1	−1	−1	0	0	0	0	6
χ_{10}	−1	−1	1	1	0	0	0	0	7
χ_{11}	1	1	1	1	0	0	0	0	8
χ_{12}	1	1	−1	−1	0	0	0	0	9
χ_{13}	0	0	0	0	0	0	0	0	10
χ_{14}	0	0	0	0	0	0	0	0	11
χ_{15}	$\sqrt{3}$i	$-\sqrt{3}$i	$\sqrt{3}$i	$-\sqrt{3}$i	0	0	0	0	12
χ_{16}	$-\sqrt{3}$i	$\sqrt{3}$i	$-\sqrt{3}$i	$\sqrt{3}$i	0	0	0	0	12
χ_{17}	$-\sqrt{3}$i	$\sqrt{3}$i	$\sqrt{3}$i	$-\sqrt{3}$i	0	0	0	0	13
χ_{18}	$\sqrt{3}$i	$-\sqrt{3}$i	$-\sqrt{3}$i	$\sqrt{3}$i	0	0	0	0	13
^	6	6	6	6	4	4	4	4	

	Classes	H	G/H	∩	
1	1+2+3+4+5+6+7+8+9+10+11+12+13+14	$\mathbf{6 \times 2^2}$	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 15 + 16	$\mathbf{D_{3,4} \times 2}$	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 17 + 18	$\mathbf{D_{3,4} \times 2}$	**2**	3	
4	1 + 2 + 5 + 6 + 9 + 11 + 12	$\mathbf{6 \times 2}$	**4**	4	
5	1 + 2 + 5 + 6 + 10 + 13 + 14	$\mathbf{6 \times 2}$	**4**	5	
6	1 + 2 + 3 + 4 + 9 + 10	$\mathbf{2^3}$	$\mathbf{D_8}$	6	
7	1 + 2 + 3 + 4	$\mathbf{2^2}$	$\mathbf{D_6 \times 2}$	7	**Z = Φ = $\mathfrak{A}$**
8	1 + 2 + 9	$\mathbf{2^2}$	$\mathbf{D_{3,4}}$	8	
9	1 + 2 + 10	$\mathbf{2^2}$	$\mathbf{D_{3,4}}$	9	
10	1 + 3 + 5 + 7	**6**	$\mathbf{D_8}$	10	
11	1 + 4 + 5 + 8	**6**	$\mathbf{D_8}$	11	
12	1 + 3	**2**	$\mathbf{D_8 \times 3}$	12	
13	1 + 4	**2**	$\mathbf{D_8 \times 3}$	13	
14	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	$\mathbf{6 \times 2}$	$\mathbf{2^2}$	1∩2	
15	1 + 2 + 5 + 6	**6**	$\mathbf{4 \times 2}$	2∩4	**G′**
16	1 + 2	**2**	$\mathbf{D_{3,4} \times 2}$	4∩9	
17	1 + 5	**3**	**16.09**	4∩10	

Sylow subgroups : [**16.09**] × 3, [**3**]
Maximal subgroups: [$\mathbf{6 \times 2^2}$], [$\mathbf{D_{3,4} \times 2}$] × 2, [**16.09**] × 3

48.28 = $\langle A^2, B^3, C^4, D^2, DB = B^{-1}D, DC = ACD \rangle$

CC	1	2	3	4	5	6	7	8	9	10
elts	1	A	C^2	AC^2	B B^2	AB AB^2	BC^2 B^2C^2	ABC^2 AB^2C^2	C AC	C^3 AC^3

CC	11	12	13	14	15	16	17	18
elts	BC ABC	B^2C AB^2C	BC^3 AB^2C^3	B^2C^3 ABC^3	B^nD AB^nD	B^nC^2D AB^nC^2D	B^nCD AB^nCD	B^nC^3D AB^nC^3D

C	1	2	3	4	5	6	7	8	9	10	
#	1	1	1	1	2	2	2	2	2	2	K
χ_1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	1
χ_3	1	1	–1	–1	1	1	–1	–1	i	–i	2
χ_4	1	1	–1	–1	1	1	–1	–1	–i	i	2
χ_5	1	1	–1	–1	1	1	–1	–1	–i	i	3
χ_6	1	1	–1	–1	1	1	–1	–1	i	–i	3
χ_7	1	1	1	1	1	1	1	1	–1	–1	4
χ_8	1	1	1	1	1	1	1	1	–1	–1	5
χ_9	2	2	2	2	–1	–1	–1	–1	2	2	6
χ_{10}	2	2	2	2	–1	–1	–1	–1	–2	–2	7
χ_{11}	2	2	–2	–2	–1	–1	1	1	2i	–2i	8
χ_{12}	2	2	–2	–2	–1	–1	1	1	–2i	2i	8
χ_{13}	2	–2	2	–2	2	–2	2	–2	0	0	9
χ_{14}	2	–2	–2	2	2	–2	–2	2	0	0	10
χ_{15}	2	–2	2	–2	–1	1	–1	1	0	0	11
χ_{16}	2	–2	2	–2	–1	1	–1	1	0	0	11
χ_{17}	2	–2	–2	2	–1	1	1	–1	0	0	12
χ_{18}	2	–2	–2	2	–1	1	1	–1	0	0	12
^	1	2	2	2	3	6	6	6	4	4	

C	11	12	13	14	15	16	17	18	
#	2	2	2	2	6	6	6	6	**K**
χ_1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	–1	–1	–1	–1	1
χ_3	i	i	–i	–i	1	–1	i	–i	2
χ_4	–i	–i	i	i	1	–1	–i	i	2
χ_5	i	i	–i	–i	–1	1	–i	i	3
χ_6	–i	–i	i	i	–1	1	i	–i	3
χ_7	–1	–1	–1	–1	1	1	–1	–1	4
χ_8	–1	–1	–1	–1	–1	–1	1	1	5
χ_9	–1	–1	–1	–1	0	0	0	0	6
χ_{10}	1	1	1	1	0	0	0	0	7
χ_{11}	–i	–i	i	i	0	0	0	0	8
χ_{12}	i	i	–i	–i	0	0	0	0	8
χ_{13}	0	0	0	0	0	0	0	0	9
χ_{14}	0	0	0	0	0	0	0	0	10
χ_{15}	$\sqrt{3}$ i	$-\sqrt{3}$ i	$\sqrt{3}$ i	$-\sqrt{3}$ i	0	0	0	0	11
χ_{16}	$-\sqrt{3}$ i	$\sqrt{3}$ i	$-\sqrt{3}$ i	$\sqrt{3}$ i	0	0	0	0	11
χ_{17}	$-\sqrt{3}$	$\sqrt{3}$	$\sqrt{3}$	$-\sqrt{3}$	0	0	0	0	12
χ_{18}	$\sqrt{3}$	$-\sqrt{3}$	$-\sqrt{3}$	$\sqrt{3}$	0	0	0	0	12
^	**12**	**12**	**12**	**12**	**2**	**2**	**4**	**4**	

	Classes	H	G/H	∩	
1	1+2+ 3+4+5+6+7+8+9+10+11+12+13+14	12×2	2	1	
2	1 + 2 + 5 + 6 + 15	$\mathbf{D}_6 \times 2$	4	2	**G′**
3	1 + 2 + 5 + 6 + 16	$\mathbf{D}_6 \times 2$	4	3	
4	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 15 + 16	$\mathbf{D}_6 \times 2^2$	2	4	
5	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 17 + 18	$\mathbf{D}_{3,4} \times 2$	2	5	
6	1 + 2 + 3 + 4 + 9 + 10	4×2	$\mathbf{D}_6$	6	
7	1 + 2 + 3 + 4 + 11 + 12 + 13 + 14			7	
8	1 + 2	2	$\mathbf{D}_6 \times 4$	8	
9	1 + 3 + 5 + 7	6	$\mathbf{D}_8$	9	
10	1 + 4 + 5 + 8	6	$\mathbf{D}_8$	10	
11	1 + 3	2	$\mathbf{D}_{12} \times 2$	11	
12	1 + 4	2	24.11	12	
13	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	6×2	2^2	1∩4	
14	1 + 5	3	16.09	2∩9	
15	1 + 2 + 3 + 4	2^2	$\mathbf{D}_6 \times 2$	4∩6	**Z = Φ = $\mathfrak{R}$**

Sylow subgroups: [**16.09**] × 3, **[3]**
Maximal subgroups: [**12 × 2**], [$\mathbf{D_6 \times 2^2}$], [$\mathbf{D_{3,4} \times 2}$], [**16.09**] × 3

48.29 = ⟨A^4, B^4, C^3, BA = $A^{-1}B$, CA = $A^{-1}C$⟩

CC	1	2	3	4	5	6	7	8	9	10
elts	1	A^2	B^2	A^2B^2	C C^2	A^2C A^2C^2	B^2C B^2C^2	A^2B^2C $A^2B^2C^2$	B A^2B	B^3 A^2B^3

CC	11	12	13	14	15	16	17	18
elts	BC A^2BC^2	B^2C A^2BC	B^3C A^2BC^2	B^3C^2 A^2B^3C	AC^n A^3C^n	AB^2C^n $A^3B^2\ C^n$	ABC^n A^3BC^n	AB^3C^n $A^3B^3C^n$

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
#	1	1	1	1	2	2	2	2	2	2	2	2	2	2	6	6	6	6	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	–1	–1	1	1	–1	–1	i	–i	i	i	–i	–i	1	–1	i	–i	2
χ_4	1	1	–1	–1	1	1	–1	–1	–i	i	i	i	–i	–i	–1	1	–i	i	2
χ_5	1	1	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	–1	–1	3
χ_6	1	1	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	–1	–1	1	1	4
χ_7	1	1	–1	–1	1	1	–1	–1	–i	i	–i	–i	i	i	1	–1	–i	i	5
χ_8	1	1	–1	–1	1	1	–1	–1	i	–i	–i	–i	i	i	–1	1	i	–i	5
χ_9	2	2	2	2	–1	–1	–1	–1	2	2	–1	–1	–1	–1	0	0	0	0	6
χ_{10}	2	2	2	2	–1	–1	–1	–1	–2	–2	1	1	1	1	0	0	0	0	7
χ_{11}	2	2	–2	–2	–1	–1	1	1	2i	–2i	–i	–i	i	i	0	0	0	0	8
χ_{12}	2	2	–2	–2	–1	–1	1	1	–2i	2i	i	i	–i	–i	0	0	0	0	8
χ_{13}	2	–2	2	–2	2	–2	2	–2	0	0	0	0	0	0	0	0	0	0	9
χ_{14}	2	–2	–2	2	2	–2	–2	2	0	0	0	0	0	0	0	0	0	0	10
χ_{15}	2	–2	2	–2	–1	1	–1	1	0	0	$\sqrt{3}$ i	$-\sqrt{3}$ i	$\sqrt{3}$ i	$-\sqrt{3}$ i	0	0	0	0	11
χ_{16}	2	–2	2	–2	–1	1	–1	1	0	0	$-\sqrt{3}$ i	$\sqrt{3}$ i	$-\sqrt{3}$ i	$\sqrt{3}$ i	0	0	0	0	11
χ_{17}	2	–2	–2	2	–1	1	1	–1	0	0	$-\sqrt{3}$	$\sqrt{3}$	$\sqrt{3}$	$-\sqrt{3}$	0	0	0	0	12
χ_{18}	2	–2	–2	2	–1	1	1	–1	0	0	$\sqrt{3}$	$-\sqrt{3}$	$-\sqrt{3}$	$\sqrt{3}$	0	0	0	0	12
^	1	2	2	2	3	6	6	6	4	4	12	12	12	12	4	4	4	4	

	Classes	H	G/H	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14	**12 × 2**	**2**	
2	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 15 + 16	**$D_{3,4}$ × 2**	**2**	
3	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 17 + 18	**$D_{3,4}$ × 2**	**2**	
4	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	**6 × 2**	**2^2**	
5	1 + 2 + 5 + 6 + 15	**$D_{3,4}$**	**4**	
6	1 + 2 + 5 + 6 + 16	**$D_{3,4}$**	**4**	
7	1 + 2 + 3 + 4 + 9 + 10	**4 × 2**	**D_6**	
8	1 + 2 + 5 + 6	**6**	**4 × 2**	**G′**
9	1 + 3 + 5 + 7	**6**	**D_8**	
10	1 + 4 + 5 + 8	**6**	**Q_8**	
11	1 + 2 + 3 + 4	**2^2**	**D_6 × 2**	**Z = Φ = $\mathfrak{N}$**
12	1 + 5	**3**	**$D_{4,4}$**	
13	1 + 2	**2**	**D_6 × 4**	
14	1 + 3	**2**	**24.11**	
15	1 + 4	**2**	**Q_{24}**	

Sylow subgroups: [$D_{4,4}$] × 3, [**3**] **Maximal subgroups:** [**12 × 2**], [$D_{3,4}$ **× 2**] × 2, [$D_{4,4}$] × 3

48.30 = $D_{12,4}$ = $\langle A^{12}, B^4, BA = A^{-1}B \rangle$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	A^6	B^2	A^6B^2	A^4 A^8	A^2 A^{10}	A^4B^2 A^8B^2	A^2B^2 $A^{10}B^2$	A^3 A^9	A^3B^2 A^9B^2	A A^{11}	A^5 A^7	AB^2 $A^{11}B^2$	A^5B^2 A^7B^2	A^{2n} B	$A^{2n+1}B^3$	$A^{2n}B^3$	$A^{2n+1}B^3$

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
#	1	1	1	1	2	2	2	2	2	2	2	2	2	2	6	6	6	6	$\mathcal{K}$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	–1	–1	1	1	–1	–1	1	–1	1	1	–1	–1	i	–i	i	–i	2
χ_4	1	1	–1	–1	1	1	–1	–1	1	–1	1	1	–1	–1	–i	i	–i	i	2
χ_5	1	1	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	–1	–1	3
χ_6	1	1	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	–1	–1	1	1	3
χ_7	1	1	–1	–1	1	1	–1	–1	–1	1	–1	–1	1	1	i	–i	–i	i	4
χ_8	1	1	–1	–1	1	1	–1	–1	–1	1	–1	–1	1	1	–i	i	i	–i	4
χ_9	2	2	2	2	–1	–1	–1	–1	2	2	–1	–1	–1	–1	0	0	0	0	5
χ_{10}	2	2	2	2	–1	–1	–1	–1	–2	–2	1	1	1	1	0	0	0	0	6
χ_{11}	2	2	–2	–2	–1	–1	1	1	2	–2	–1	–1	1	1	0	0	0	0	7
χ_{12}	2	2	–2	–2	–1	–1	1	1	–2	2	1	1	–1	–1	0	0	0	0	8
χ_{13}	2	–2	2	–2	2	–2	2	–2	0	0	0	0	0	0	0	0	0	0	9
χ_{14}	2	–2	–2	2	2	–2	–2	2	0	0	0	0	0	0	0	0	0	0	10
χ_{15}	2	–2	2	–2	–1	1	–1	1	0	0	$\sqrt{3}$	$-\sqrt{3}$	$\sqrt{3}$	$-\sqrt{3}$	0	0	0	0	11
χ_{16}	2	–2	2	–2	–1	1	–1	1	0	0	$-\sqrt{3}$	$\sqrt{3}$	$-\sqrt{3}$	$\sqrt{3}$	0	0	0	0	11
χ_{17}	2	–2	–2	2	–1	1	1	–1	0	0	$\sqrt{3}$	$-\sqrt{3}$	$-\sqrt{3}$	$\sqrt{3}$	0	0	0	0	12
χ_{18}	2	–2	–2	2	–1	1	1	–1	0	0	$-\sqrt{3}$	$\sqrt{3}$	$\sqrt{3}$	$-\sqrt{3}$	0	0	0	0	12
^	1	2	2	2	3	6	6	6	4	4	12	12	12	12	4	4	4	4	

	Classes	H	G/H	∩	
1	1+2+3+4+5+6+7+8+9+10+11+12+13+14	**12 × 2**	**2**	1	
2	1 + 2 + 5 + 6 + 9 + 11 + 12	**12**	**4**	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 17 + 18	**$D_{3,4}$ × 2**	**2**	3	
4	1 + 2 + 5 + 6 + 10 + 13 + 14	**12**	**2^2**	4	
5	1 + 2 + 3 + 4 + 9 + 10	**4 × 2**	**D_6**	5	
6	1 + 2 + 3 + 4	**2^2**	**D_{12}**	6	**Z = Φ = $\mathfrak{A}$**
7	1 + 2 + 9	**4**	**$D_{3,4}$**	7	
8	1 + 2 + 10	**4**	**$D_{3,4}$**	8	
9	1 + 3 + 5 + 7	**6**	**D_8**	9	
10	1 + 4 + 5 + 8	**6**	**Q_8**	10	
11	1 + 3	**2**	**D_{24}**	11	
12	1 + 4	**2**	**Q_{24}**	12	
13	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	**6 × 2**	**2^2**	1∩3	
14	1 + 2 + 5 + 6	**6**	**4 × 2**	2∩3	**G′**
15	1 + 2	**2**	**$D_{6,4}$**	2∩6	
16	1 + 5	**3**	**$D_{4,4}$**	2∩9	

Sylow subgroups: [$D_{4,4}$] × 3, [**3**] **Maximal subgroups:** [**12 × 2**], [$D_{3,4}$ **× 2**] × 2, [$D_{4,4}$] × 3

48.31 = ⟨A^4, B^3, C^2, D^2, $DB = B^{-1}D$, $DC = A^2CD$⟩

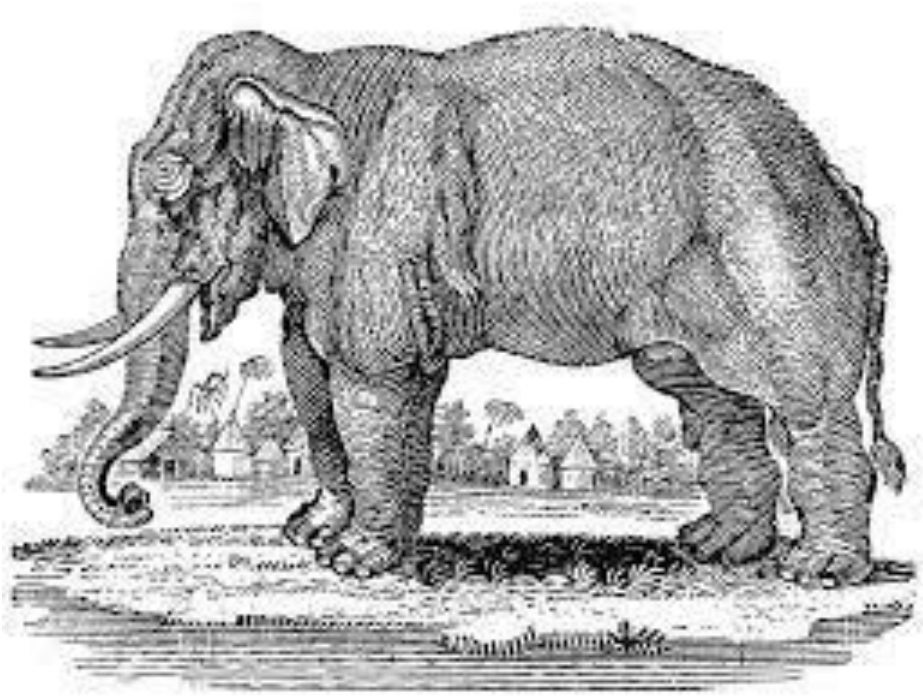
CC	1	2	3	4	5	6	7	8	9	10	11	12
elts	1	A^2	A	A^3	B B^2	A^2B AB^2	AB A^2B^2	A^3B A^3B^2	C A^2C	AC A^3C	BC A^2B^2C	B^2C A^2BC

CC	13	14	15	16	17	18
elts	ABC A^3B^2C	AB^2C A^3BC	B^nD A^2B^nD	AB^nD A^3B^nD	B^nCD A^2B^nCD	AB^nCD A^3B^nCD

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
#	1	1	1	1	2	2	2	2	2	2	2	2	2	2	6	6	6	6	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	–1	–1	2
χ_4	1	1	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	–1	–1	1	1	3
χ_5	1	1	–1	–1	1	1	–1	–1	1	–1	1	1	–1	–1	1	–1	1	–1	4
χ_6	1	1	–1	–1	1	1	–1	–1	1	–1	1	1	–1	–1	–1	1	–1	1	5
χ_7	1	1	–1	–1	1	1	–1	–1	–1	1	–1	–1	1	1	1	–1	–1	1	6
χ_8	1	1	–1	–1	1	1	–1	–1	–1	1	–1	–1	1	1	–1	1	1	–1	7
χ_9	2	2	2	2	–1	–1	–1	–1	2	2	–1	–1	–1	–1	0	0	0	0	8
χ_{10}	2	2	2	2	–1	–1	–1	–1	–2	–2	1	1	1	1	0	0	0	0	9
χ_{11}	2	2	–2	–2	–1	–1	1	1	2	–2	–1	–1	1	1	0	0	0	0	10
χ_{12}	2	2	–2	–2	–1	–1	1	1	–2	2	1	1	–1	–1	0	0	0	0	11
χ_{13}	2	–2	2i	–2i	2	–2	2i	–2i	0	0	0	0	0	0	0	0	0	0	12
χ_{14}	2	–2	–2i	2i	2	–2	–2i	2i	0	0	0	0	0	0	0	0	0	0	12
χ_{15}	2	–2	2i	–2i	–1	1	–i	i	0	0	i	–i	–1	1	0	0	0	0	0
χ_{16}	2	–2	–2i	2i	–1	1	i	–i	0	0	–i	i	–1	1	0	0	0	0	0
χ_{17}	2	–2	2i	–2i	–1	1	–i	i	0	0	–i	i	1	–1	0	0	0	0	0
χ_{18}	2	–2	–2i	2i	–1	1	i	–i	0	0	i	–i	1	–1	0	0	0	0	0
^	1	2	4	4	3	6	12	12	2	4	6	6	12	12	2	4	4	4	

	Classes	H	G/H	∩	
1	1+2+3+4+5+6+7+8+9+10+11+12+13+14	12×2	2	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 15 + 16	$\mathbf{D}_6 \times 4$	2	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 17 + 18			3	
4	1 + 2 + 5 + 6 + 9 + 11 + 12 + 15 + 17	24.11	2	4	
5	1 + 2 + 5 + 6 + 9 + 11 + 12 + 16 + 18	$\mathbf{D}_{3,4} \times 2$	2	5	
6	1 + 2 + 5 + 6 + 10 + 13 + 14 + 15 + 18	$\mathbf{D}_6 \times 4$	2	6	
7	1 + 2 + 5 + 6 + 10 + 13 + 14 + 16 + 17	$\mathbf{Q}_{24}$	2	7	
8	1 + 2 + 3 + 4 + 9 + 10	4×2	D_6	8	
9	1 + 2 + 3 + 4	4	$\mathbf{D}_6 \times 2$	9	$\mathbf{Z} = \mathfrak{N}$
10	1 + 2 + 9	2^2	$\mathbf{D}_6 \times 2$	10	
11	1 + 2 + 10	4	$\mathbf{D}_6 \times 2$	11	
12	1 + 5	3	16.08	12	
13	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	12	2^2	1∩2	
14	1 + 2 + 5 + 6 + 9 + 11 + 12	6×2	2^2	1∩4	
15	1 + 2 + 5 + 6	6	2^3	2∩4	$\mathbf{G}'$
16	1 + 2 + 5 + 6 + 15	$\mathbf{D}_6 \times 2$	2^2	2∩6	
17	1 + 2 + 5 + 6 + 16	$\mathbf{D}_{3,4}$	2^2	2∩7	
18	1 + 2	2	$\mathbf{D}_6 \times 2^2$	2∩10	$\mathbf{\Phi}$
19	1 + 2 + 5 + 6 + 17	$\mathbf{D}_{3,4}$	2^2	3∩4	
20	1 + 2 + 5 + 6 + 18	$\mathbf{D}_{3,4}$	2^2	3∩5	
21	1 + 2 + 5 + 6 + 10 + 13 + 14	12	2^2	6∩7	

Sylow subgroups: [**16.08**] × 3, [**3**]
Maximal subgroups: [$\mathbf{6 \times 2^2}$], [$\mathbf{D_6 \times 4}$] × 2, [**24.11**] × 2, [$\mathbf{D_{24}}$], [$\mathbf{Q_{24}}$], [**16.08**] × 3



48.32 = ⟨A^3, B^8, C^2, BA = $A^{-1}B$, CB = B^5C⟩

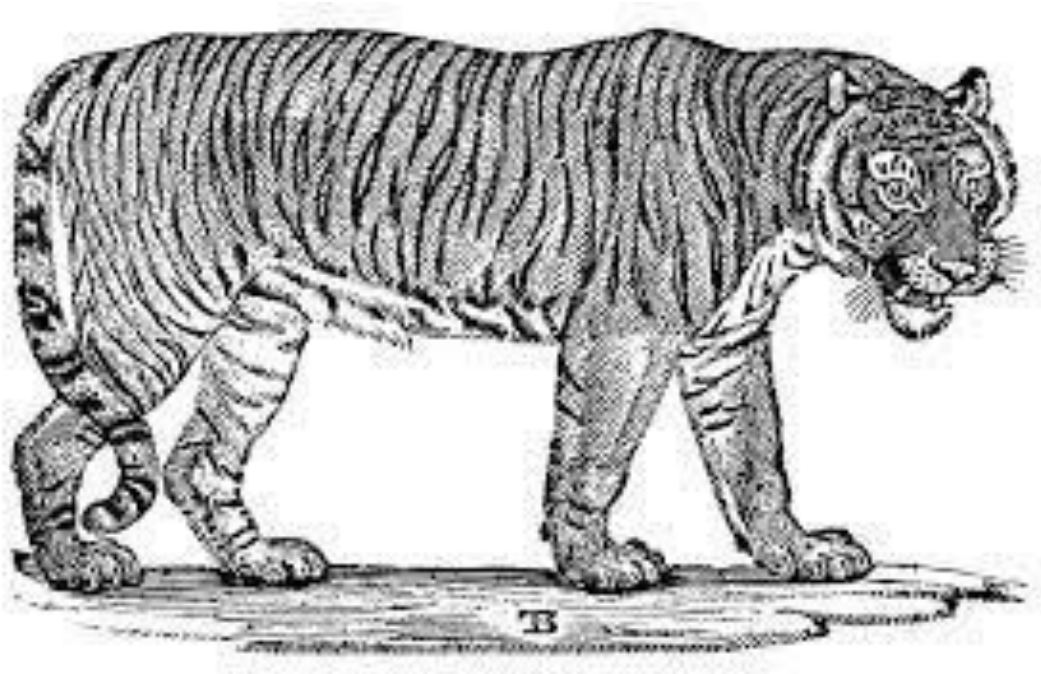
CC	1	2	3	4	5	6	7	8	9	10
elts	1	B^4	B^2	B^6	A A^2	AB^4 A^2B^4	AB^2 A^2B^2	AB^6 A^2B^6	C B^4C	B^2C B^6C

CC	11	12	13	14	15	16	17	18
elts	AC A^2B^4C	A^2C AB^4C	AB^2C A^2B^6C	A^2B^2C AB^6C	A^nB A^nB^5	A^nB^3 A^nB^7	A^nBC A^nB^5C	A^nB^3C A^nB^7C

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
#	1	1	1	1	2	2	2	2	2	2	2	2	2	2	6	6	6	6	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	–1	–1	2
χ_4	1	1	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	–1	–1	1	1	3
χ_5	1	1	–1	–1	1	1	–1	–1	1	–1	1	1	–1	–1	i	–i	i	–i	4
χ_6	1	1	–1	–1	1	1	–1	–1	1	–1	1	1	–1	–1	–i	i	–i	i	4
χ_7	1	1	–1	–1	1	1	–1	–1	–1	1	–1	–1	1	1	i	–i	–i	i	5
χ_8	1	1	–1	–1	1	1	–1	–1	–1	1	–1	–1	1	1	–i	i	i	–i	5
χ_9	2	2	2	2	–1	–1	–1	–1	2	2	–1	–1	–1	–1	0	0	0	0	6
χ_{10}	2	2	2	2	–1	–1	–1	–1	–2	–2	1	1	1	1	0	0	0	0	7
χ_{11}	2	2	–2	–2	–1	–1	1	1	2	–2	–1	–1	1	1	0	0	0	0	8
χ_{12}	2	2	–2	–2	–1	–1	1	1	–2	2	1	1	–1	–1	0	0	0	0	9
χ_{13}	2	–2	2i	–2i	2	–2	2i	–2i	0	0	0	0	0	0	0	0	0	0	10
χ_{14}	2	–2	–2i	2i	2	–2	–2i	2i	0	0	0	0	0	0	0	0	0	0	10
χ_{15}	2	–2	2i	–2i	–1	1	–i	i	0	0	i	–i	–1	1	0	0	0	0	0
χ_{16}	2	–2	–2i	2i	–1	1	i	–i	0	0	–i	i	–1	1	0	0	0	0	0
χ_{17}	2	–2	2i	–2i	–1	1	–i	i	0	0	–i	i	1	–1	0	0	0	0	0
χ_{18}	2	–2	–2i	2i	–1	1	i	–i	0	0	i	–i	1	–1	0	0	0	0	0
^	1	2	4	4	3	6	12	12	2	4	6	6	12	12	8	8	8	8	

	Classes	H	G/H	∩	
1	1+2+3+4+5+6+7+8+9+10+11+12+13+14	**12 × 2**	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 15 + 16	$\mathbf{D_{3,8}}$	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 17 + 18	$\mathbf{D_{3,8}}$	**2**	3	
4	1 + 2 + 5 + 6 + 9 + 11 + 12	**6 × 2**	**4**	4	
5	1 + 2 + 5 + 6 + 10 + 13 + 14	**12**	**4**	5	
6	1 + 2 + 3 + 4 + 9 + 10	**4 × 2**	$\mathbf{D_6}$	6	$\mathbf{Z_2} = \mathfrak{A}$
7	1 + 2 + 3 + 4	**4**	$\mathbf{D_6 \times 2}$	7	$\mathbf{Z} = \mathbf{\Phi}$
8	1 + 2 + 9	$\mathbf{2^2}$	$\mathbf{D_{3,4}}$	8	
9	1 + 2 + 10	**4**	$\mathbf{D_6 \times 2}$	9	
10	1 + 5	**3**	$\mathbf{M_{8,2}^{(3)}}$	10	
11	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	**12**	$\mathbf{2^2}$	1∩2	
12	1 + 2 + 5 + 6	**6**	**4 × 2**	2∩4	**G′**
13	1 + 2	**2**	**24.08**	2∩8	

Sylow subgroups: $[\mathbf{M_{8,2}^{(3)}}] \times \mathbf{3}$, $[\mathbf{3}]$ **Maximal subgroups:** $[\mathbf{12 \times 2}]$, $[\mathbf{D_{3,8}}] \times 2$, $[\mathbf{M_{8,2}^{(5)}}] \times 3$



48.33 = $\langle A^3, B^8, C^2, CB = B^5C, CA = A^{-1}C \rangle$ = $M_{24,2}^{(5)}$

CC	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
elts	1	B^4	B^2	B^6	A A^2	AB^4 A^2B^4	AB^2 A^2B^2	AB^6 A^2B^6	B B^5	B^3 B^7	AB A^2B^5	A^2B AB^5	AB^3 A^2B^7	A^2B^3 AB^7	A^nC A^nB^4C	A^nB^2C A^nB^6C	A^nBC A^nB^5C	A^nB^3C A^nB^7C

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
#	1	1	1	1	2	2	2	2	2	2	2	2	2	2	6	6	6	6	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	–1	–1	1	1	–1	–1	i	–i	i	i	–i	–i	1	–1	i	–i	2
χ_4	1	1	–1	–1	1	1	–1	–1	i	–i	i	i	–i	–i	–1	1	–i	i	3
χ_5	1	1	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	–1	–1	4
χ_6	1	1	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	–1	–1	1	1	5
χ_7	1	1	–1	–1	1	1	–1	–1	–i	i	–i	–i	i	i	1	–1	–i	i	2
χ_8	1	1	–1	–1	1	1	–1	–1	–i	i	–i	–i	i	i	–1	1	i	–i	3
χ_9	2	2	2	2	–1	–1	–1	–1	2	2	–1	–1	–1	–1	0	0	0	0	6
χ_{10}	2	2	2	2	–1	–1	–1	–1	–2	–2	1	1	1	1	0	0	0	0	7
χ_{11}	2	2	–2	–2	–1	–1	1	1	2i	–2i	–i	–i	i	i	0	0	0	0	8
χ_{12}	2	2	–2	–2	–1	–1	1	1	–2i	2i	i	i	–i	–i	0	0	0	0	8
χ_{13}	2	–2	2i	–2i	2	–2	2i	–2i	0	0	0	0	0	0	0	0	0	0	9
χ_{14}	2	–2	–2i	2i	2	–2	–2i	2i	0	0	0	0	0	0	0	0	0	0	9
χ_{15}	2	–2	–2i	2i	–1	1	i	–i	0	0	1+i	–1–i	–1	1	0	0	0	0	0
χ_{16}	2	–2	2i	–2i	–1	1	–i	i	0	0	1–i	–1+i	–1	1	0	0	0	0	0
χ_{17}	2	–2	2i	–2i	–1	1	–i	i	0	0	–1+i	1–i	–1–i	1+i	0	0	0	0	0
χ_{18}	2	–2	–2i	2i	–1	1	i	–i	0	0	–1–i	1+i	–1+i	1–i	0	0	0	0	0
^	1	2	4	4	3	6	12	12	8	8	24	24	24	24	2	4	8	8	

	Classes	H	G/H	∩	
1	1+2+3+4+5+6+7+8+9+10+11+12+13+14	**24**	**2**	1	
2	1 + 2 + 5 + 6 + 15	**$D_6 \times 2$**	**4**	2	
3	1 + 2 + 5 + 6 + 16	**$D_{3,4}$**	**4**	3	
4	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 15 + 16	**$D_6 \times 4$**	**2**	4	
5	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 17 + 18	**$D_{3,8}$**	**2**	5	
6	1 + 2 + 3 + 4 + 9 + 10	**8**	**D_6**	6	
7	1 + 2 + 3 + 4	**4**	**$D_6 \times 2$**	7	**Z = Φ = ℜ**
8	1 + 2	**2**	**$D_6 \times 4$**	8	
9	1 + 5	**3**	**$M_{8,2}^{(3)}$**	9	
10	1 + 2 + 5 + 6	**6**	**4×2**	1∩2	**G′**
11	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	**12**	**2^2**	1∩4	

Sylow subgroups: [$M_{8,2}^{(3)}$] × 3, [**3**]

Maximal subgroups: [**24**], [$D_6 \times 4$], [$D_{3,8}$], [$M_{8,2}^{(3)}$] × 3

48.34 = $SL_{2,3} \times 2 = \langle A^4, B^2 = A^2, C^3, D^2, BA = A^{-1}B, CA = A^{-1}BC, CB = A^{-1}C \rangle$

48.35 = $\langle A^4, B^2 = A^2, C^2 = A^2, D^3, CB = A^2BC, DB = BCD, DC = BD \rangle$

CC	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6	Γ_7	Γ_8	Γ_9	Γ_{10}	Γ_{11}	Γ_{12}	Γ_{13}	Γ_{14}
elts	1	A^2	A	A^3	B	AB	D	A^2D	AD	A^3D	D^2	A^2D^2	AD^2	A^3D^2
					C	AC	BCD	BD	ABCD	A^2BCD	A^2BD^2	A^2BCD^2	A^3BD^2	ABD^2
					BC	ABC	A^2BD	CD	A^3BD	ABD	CD^2	A^2CD	ACD^2	$ABCD^2$
					A^2B	A^3B	A^2CD	A^2BCD	A^3CD	ACD	BCD^2	BD^2	$ABCD^2$	A^3BCD^2
					A^2C	A^3C								
					A^2BC	A^3BC								

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
#	1	1	1	1	6	6	4	4	4	4	4	4	4	4	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	–1	–1	1	–1	1	1	–1	–1	1	1	–1	–1	1
χ_3	1	1	1	1	1	1	ω	ω	ω	ω	ω^2	ω^2	ω^2	ω^2	2
χ_4	1	1	1	1	1	1	ω^2	ω^2	ω^2	ω^2	ω	ω	ω	ω	2
χ_5	1	1	–1	–1	1	–1	ω	ω	–ω	–ω	ω^2	ω^2	$-\omega^2$	$-\omega^2$	3
χ_6	1	1	–1	–1	1	–1	ω^2	ω^2	$-\omega^2$	$-\omega^2$	ω	ω	–ω	–ω	3
χ_7	2	–2	2i	–2i	0	0	–1	–1	–i	i	–1	–1	–i	i	0
χ_8	2	–2	–2i	2i	0	0	–1	1	i	–i	–1	1	i	–i	0
χ_9	2	–2	2i	–2i	0	0	–ω	ω	–iω	iω	$-\omega^2$	ω^2	$-i\omega^2$	$i\omega^2$	0
χ_{10}	2	–2	–2i	2i	0	0	–ω	ω	iω	–iω	$-\omega^2$	ω^2	$i\omega^2$	$-i\omega^2$	0
χ_{11}	2	–2	2i	–2i	0	0	$-\omega^2$	ω^2	$-i\omega^2$	$i\omega^2$	–ω	ω	–iω	iω	0
χ_{12}	2	–2	–2i	2i	0	0	$-\omega^2$	ω^2	$i\omega^2$	$-i\omega^2$	–ω	ω	iω	–iω	0
χ_{13}	3	3	3	3	–1	–1	0	0	0	0	0	0	0	0	4
χ_{14}	3	3	–3	–3	–1	1	0	0	0	0	0	0	0	0	5
^	1	2	4	4	4	4	3	6	12	12	3	6	12	12	

	Classes	H	G/H	∩	
1	1 + 2 + 5 + 7 + 8 + 11 + 12	$SL_{2,3}$	2	1	
2	1 + 2 + 3 + 4 + 5 + 6	D_{16}	3	2	
3	1 + 2 + 5	Q_8	6	3	G′
4	1 + 2 + 3 + 4	4	A_4	4	Z = 𝔄
5	1 + 2	2	$A_4 \times 2$	5	Φ

Sylow subgroups: [**16.08**], [**3**] × 4 **Maximal subgroups:** [$SL_{2,3}$], [**16.08**], [**12**] × 4

48.36 = $S_4 \times 2$

= ⟨A^2, B^2, C^3, D^4, E^2, CA = ABC, CB = AC, DA = BD, DB = AD, DC = C^{-1}D⟩

48.37 = ⟨A^2, B^2, C^3, D^4, CA = ABC, CB = AC, DA = BD, DB = AD, DC = C^{-1}D⟩

CC	1	2	3	4	5	6	7	8	9	10
elts	1	D^2	A B AB	AD^2 BD^2 ABD^2	A^mB^nC $A^mB^nC^2$	$A^mB^nCD^2$ $A^mB^nC^2D^2$	D BD C^2D BC^2D CD ACD	D^3 BD^3 C^2D^3 BC^2D^3 CD^3 ACD^3	BD AD ABC^2D BC^2D^3 CD^3 ACD^3	CD ACD BC^2D BD^3 AD^3 ABC^2D^3

C	1	2	3	4	5	6	7	8	9	10	
#	1	1	3	3	8	8	6	6	6	6	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	–1	1	–1	1	–1	i	–i	i	–i	2
χ_4	1	–1	1	–1	1	–1	–i	i	–i	i	2
χ_5	2	2	2	2	–1	–1	0	0	0	0	3
χ_6	2	–2	2	–2	–1	1	0	0	0	0	4
χ_7	3	3	–1	–1	0	0	–1	–1	–i	i	5
χ_8	3	3	–1	–1	0	0	–1	1	i	–i	5
χ_9	3	–3	–1	1	0	0	i	–i	i	–i	0
χ_{10}	3	–3	–1	1	0	0	–i	i	–i	i	0
^	1	2	2	2	3	6	4	4	4	4	

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6	**$A_4 \times 2$**	**2**	1	
2	1 + 3 + 5	**A_4**	**2^2**	2	**G′**
3	1 + 2 + 3 + 4	**2^3**	**D_6**	3	
4	1 + 3	**2^2**	**$D_{3,4}$**	4	
5	1 + 2	**2**	**$A_4 \times 2$**	5	**Z = Φ = ℜ**

Sylow subgroups: [**16.09**] × 3, [**3**] × 4
Maximal subgroups: [$A_4 \times 2$], [**16.09**] × 3, [$D_{3,4}$] × 4

48.38 = $D_8 \times D_6$ = ⟨A^4, B^2, C^3, D^2, BA = A^{-1}B, DC = C^{-1}D⟩

48.39 = $Q_8 \times D_6$ = ⟨A^4, $B^2 = A^2$, C^3, D^2, BA = A^{-1}B, DC = C^{-1}D⟩

48.40 = ⟨A^4, B^2, C^3, $D^2 = A^2$, $BA = A^{-1}B$, $DC = C^{-1}D$⟩

CC	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
elts	1	A^2	A A^3	B A^2B	AB A^3B	C C^2	A^2C A^2C^2	AC A^3C AC^2 A^3C^2	BC A^2BC BC^2 A^2BC^2	ABC A^3BC ABC^2 A^3BC^2	D CD C^2D	A^2D A^2CD A^2C^2D	AC^nD A^3C^nD	BC^nD A^2BC^nD	ABC^nD A^3BC^nD

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
#	1	1	2	2	2	2	2	4	4	4	3	3	6	6	6	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	–1	1
χ_3	1	1	1	–1	–1	1	1	1	–1	–1	1	1	1	–1	–1	2
χ_4	1	1	1	–1	–1	1	1	1	–1	–1	–1	–1	–1	1	1	3
χ_5	1	1	–1	1	–1	1	1	–1	1	–1	1	1	–1	1	–1	4
χ_6	1	1	–1	1	–1	1	1	–1	1	–1	–1	–1	1	–1	1	5
χ_7	1	1	–1	–1	1	1	1	–1	–1	1	1	1	–1	–1	1	6
χ_8	1	1	–1	–1	1	1	1	–1	–1	1	–1	–1	1	1	–1	7
χ_9	2	2	2	2	2	–1	–1	–1	–1	–1	0	0	0	0	0	8
χ_{10}	2	2	2	–2	–2	–1	–1	–1	1	1	0	0	0	0	0	9
χ_{11}	2	2	–2	2	–2	–1	–1	1	–1	1	0	0	0	0	0	10
χ_{12}	2	2	–2	–2	2	–1	–1	1	1	–1	0	0	0	0	0	11
χ_{13}	2	–2	0	0	0	2	–2	0	0	0	2i	–2i	0	0	0	12
χ_{14}	2	–2	0	0	0	2	–2	0	0	0	–2i	2i	0	0	0	12
χ_{15}	4	–4	0	0	0	–2	2	0	0	0	0	0	0	0	0	0
^	1	2	4	2	2	3	6	12	6	6	4	4	4	4	4	

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10	$\mathbf{D_8 \times 3}$	**2**	1	
2	1 + 2 + 3 + 6 + 7 + 8 + 11 + 12 + 13	$\mathbf{Q_{24}}$	**2**	2	
3	1 + 2 + 3 + 6 + 7 + 8 + 14 + 15	$\mathbf{Q_{24}}$	**2**	3	
4	1 + 2 + 4 + 6 + 7 + 9 + 11 + 12 + 14	$\mathbf{D_{3,4} \times 2}$	**2**	4	
5	1 + 2 + 4 + 6 + 7 + 9 + 13 + 15	$\mathbf{D_{3,4} \times 2}$	**2**	5	
6	1 + 2 + 5 + 6 + 7 + 10 + 11 + 12 + 15	$\mathbf{D_{3,4} \times 2}$	**2**	6	
7	1 + 2 + 5 + 6 + 7 + 10 + 13 + 14	$\mathbf{D_{3,4}}$ **2**	**2**	7	
8	1 + 2 + 3 + 4 + 5	$\mathbf{D_8}$	$\mathbf{D_6}$	8	
9	1 + 2 + 3	**4**	$\mathbf{D_6 \times 2}$	9	
10	1 + 2 + 4	$\mathbf{2^2}$	$\mathbf{D_6 \times 2}$	10	
11	1 + 2 + 5	$\mathbf{2^2}$	$\mathbf{D_6 \times 2}$	11	
12	1 + 6	**3**	**16.08**	12	
13	1 + 2 + 3 + 6 + 7 + 8	**12**	$\mathbf{2^2}$	1∩2	
14	1 + 2 + 4 + 6 + 7 + 9	$\mathbf{6 \times 2}$	$\mathbf{2^2}$	1∩4	
15	1 + 2 + 5 + 6 + 7 + 10	$\mathbf{6 \times 2}$	$\mathbf{2^2}$	1∩6	
16	1 + 2 + 6 + 7 + 11 + 12	$\mathbf{D_{3,4}}$	$\mathbf{2^2}$	2∩4	
17	1 + 2 + 6 + 7 + 13	$\mathbf{D_{3,4}}$	$\mathbf{2^2}$	2∩5	
18	1 + 2	**2**	$\mathbf{D_6 \times 2^2}$	2∩10	$\mathbf{Z = \Phi = \mathfrak{N}}$
19	1 + 2 + 6 + 7 + 14	$\mathbf{D_{3,4}}$	$\mathbf{2^2}$	3∩4	
20	1 + 2 + 6 + 7 + 15	$\mathbf{D_{3,4}}$	$\mathbf{2^2}$	3∩5	
21	1 + 2 + 6 + 7	**6**	$\mathbf{2^3}$	13∩14	$\mathbf{G'}$

Sylow subgroups: [**16.08**] × 3, [**3**]
Maximal subgroups: [$\mathbf{D_8 \times 3}$], [$\mathbf{D_6 \times 4}$], [$\mathbf{D_{3,4} \times 2}$] × 2, [**24.11**] × 2, [$\mathbf{Q_{24}}$], [**16.08**] × 3

48.41 = ⟨A^4, $B^2 = A^2$, C^3, D^2, $= A^2$, $BA = A^{-1}B$, $DC = ACD$⟩

CC	1	2	3	4	5	6	7	8	9	10
elts	1	A^2	A A^3	B A^2B	AB A^3B	C C^2	A^2C A^2C^2	AC A^3C AC^2 A^3C^2	BC A^2BC BC^2 A^2BC^2	ABC A^3BC ABC^2 A^3BC^2

CC	11	12	13	14	15
elts	D C^2D A^2CD	CD A^2D A^2C^2D	AC^nD A^3C^nD	BC^nD A^2BC^nD	ABC^nD A^3BC^nD

C	1	2	3	4	5	6	7	8	9	10		11	12	13	14	15	
#	1	1	2	2	2	2	2	4	4	4		3	3	6	6	6	ℋ
χ_1	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1		–1	–1	–1	–1	–1	1
χ_3	1	1	1	–1	–1	1	1	1	–1	–1		1	1	1	–1	–1	2
χ_4	1	1	1	–1	–1	1	1	1	–1	–1		–1	–1	–1	1	1	3
χ_5	1	1	–1	1	–1	1	1	–1	1	–1		1	1	–1	1	–1	4
χ_6	1	1	–1	1	–1	1	1	–1	1	–1		–1	–1	1	–1	1	5
χ_7	1	1	–1	–1	1	1	1	–1	–1	1		1	1	–1	–1	1	6
χ_8	1	1	–1	–1	1	1	1	–1	–1	1		–1	–1	1	1	–1	7
χ_9	2	2	2	2	2	–1	–1	–1	–1	–1		0	0	0	0	0	8
χ_{10}	2	2	2	–2	–2	–1	–1	–1	1	1		0	0	0	0	0	9
χ_{11}	2	2	–2	2	–2	–1	–1	1	–1	1		0	0	0	0	0	10
χ_{12}	2	2	–2	–2	2	–1	–1	1	1	–1		0	0	0	0	0	11
χ_{13}	2	–2	0	0	0	2	–2	0	0	0		2i	–2i	0	0	0	12
χ_{14}	2	–2	0	0	0	2	–2	0	0	0		–2i	2i	0	0	0	12
χ_{15}	4	–4	0	0	0	–2	2	0	0	0		0	0	0	0	0	0
^	1	2	4	4	4	3	6	12	12	12		4	4	4	4	4	

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10	**$Q_8 \times 3$**	**2**	1	
2	1 + 2 + 3 + 6 + 7 + 8 + 11 + 12 + 13	**Q_{24}**	**2**	2	
3	1 + 2 + 3 + 6 + 7 + 8 + 14 + 15	**Q_{24}**	**2**	3	
4	1 + 2 + 4 + 6 + 7 + 9 + 11 + 12 + 14	**Q_{24}**	**2**	4	
5	1 + 2 + 4 + 6 + 7 + 9 + 13 + 15	**Q_{24}**	**2**	5	
6	1 + 2 + 5 + 6 + 7 + 10 + 11 + 12 + 15	**Q_{24}**	**2**	6	
7	1 + 2 + 5 + 6 + 7 + 10 + 13 + 14	**Q_{24}**	**2**	7	
8	1 + 2 + 3 + 4 + 5	**Q_8**	**D_6**	8	
9	1 + 2 + 3	**4**	**$D_6 \times 2$**	9	
10	1 + 2 + 4	**4**	**$D_6 \times 2$**	10	
11	1 + 2 + 5	**4**	**$D_6 \times 2$**	11	
12	1 + 6	**3**	**16.08**	12	
13	1 + 2 + 3 + 6 + 7 + 8	**12**	**2^2**	1∩2	
14	1 + 2 + 4 + 6 + 7 + 9	**12**	**2^2**	1∩4	
15	1 + 2 + 5 + 6 + 7 + 10	**12**	**2^2**	1∩6	
16	1 + 2 + 6 + 7 + 11 + 12	**$D_{3,4}$**	**2^2**	2∩6	
17	1 + 2 + 6 + 7 + 13	**$D_{3,4}$**	**2^2**	2∩7	
18	1 + 2 + 6 + 7 + 14	**$D_{3,4}$**	**2^2**	3∩4	
19	1 + 2 + 6 + 7 + 15	**$D_{3,4}$**	**2^2**	3∩5	
20	1 + 2	**2**	**$D_6 \times 2^2$**	3∩9	**$Z = \Phi = \mathfrak{N}$**
21	1 + 2 + 6 + 7	**6**	**2^3**	13∩14	**G′**

Sylow subgroups: [**16.08**] × 3, [**3**]
Maximal subgroups: [Q_8] × 3, [**$D_6 \times 4$**] × 3, [D_{24}] × 3, [**16.08**] × 3

48.42 = D_{48} = $\langle A^{24}, B^2, BA = A^{-1}B \rangle$

Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6	Γ_7	Γ_8	Γ_9	Γ_{10}	Γ_{11}	Γ_{12}	Γ_{13}	Γ_{14}	Γ_{15}
1	A^{12}	A^6 A^{18}	A^8 A^{16}	A^4 A^{20}	A^3 A^{21}	A^9 A^{15}	A^2 A^{22}	A^{10} A^{14}	A A^{23}	A^5 A^{19}	A^7 A^{17}	A^{11} A^{13}	$A^{2n}B$	$A^{2n+1}B$

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
#	1	1	2	2	2	2	2	2	2	2	2	2	2	12	12
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1
χ_3	1	1	1	1	1	−1	−1	1	1	−1	−1	−1	−1	1	−1
χ_4	1	1	1	1	1	−1	−1	1	1	−1	−1	−1	−1	−1	1
χ_5	2	2	2	−1	−1	2	2	−1	1	−1	−1	−1	−1	0	0
χ_6	2	2	2	−1	−1	−2	−2	−1	1	1	1	1	1	0	0
χ_7	2	2	−2	2	2	0	0	−2	−2	0	0	0	0	0	0
χ_8	2	−2	0	2	−2	$\sqrt{2}$	$-\sqrt{2}$	0	1	$-\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$	0	0
χ_9	2	−2	0	2	−1	$-\sqrt{2}$	$\sqrt{2}$	0	0	$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	0	0
χ_{10}	2	2	−2	−1	−1	0	0	1	1	$\sqrt{3}$	$-\sqrt{3}$	$-\sqrt{3}$	$\sqrt{3}$	0	0
χ_{11}	2	2	−2	−1	−1	0	0	1	1	$-\sqrt{3}$	$\sqrt{3}$	$\sqrt{3}$	$-\sqrt{3}$	0	0
χ_{12}	2	−2	0	−1	1	$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{3}$	$-\sqrt{3}$	$\frac{\sqrt{6}+\sqrt{2}}{2}$	$\frac{\sqrt{6}-\sqrt{2}}{2}$	$\frac{-\sqrt{6}+\sqrt{2}}{2}$	$\frac{-\sqrt{6}-\sqrt{2}}{2}$	0	0
χ_{13}	2	−2	0	−1	1	$-\sqrt{2}$	$\sqrt{2}$	$-\sqrt{3}$	$\sqrt{3}$	$\frac{\sqrt{6}-\sqrt{2}}{2}$	$\frac{\sqrt{6}+\sqrt{2}}{2}$	$\frac{-\sqrt{6}-\sqrt{2}}{2}$	$\frac{-\sqrt{6}+\sqrt{2}}{2}$	0	0
χ_{14}	2	−2	0	−1	1	$\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{3}$	$\sqrt{3}$	$\frac{-\sqrt{6}+\sqrt{2}}{2}$	$\frac{-\sqrt{6}-\sqrt{2}}{2}$	$\frac{\sqrt{6}+\sqrt{2}}{2}$	$\frac{\sqrt{6}-\sqrt{2}}{2}$	0	0
χ_{15}	2	−2	0	−1	1	$-\sqrt{2}$	$\sqrt{2}$	$\sqrt{3}$	$-\sqrt{3}$	$\frac{-\sqrt{6}-\sqrt{2}}{2}$	$\frac{-\sqrt{6}+\sqrt{2}}{2}$	$\frac{\sqrt{6}-\sqrt{2}}{2}$	$\frac{\sqrt{6}+\sqrt{2}}{2}$	0	0
^	**1**	**2**	**4**	**3**	**6**	**8**	**8**	**12**	**12**	**24**	**24**	**24**	**24**	**2**	**2**

	Classes	**H**	**G/H**	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13	24	2	
2	1 + 2 + 3 + 4 + 5 + 8 + 9 + 14	$\mathbf{D}_{12} \times 2$	2	
3	1 + 2 + 3 + 4 + 5 + 8 + 9 + 15	$\mathbf{D}_{12} \times 2$	2	
4	1 + 2 + 3 + 6 + 7	8	$\mathbf{D}_6$	
5	1 + 2 + 3 + 4 + 5 + 8 + 9	12	2^2	G′
6	1 + 2 + 4 + 5	6	$\mathbf{D}_8$	
7	1 + 2 + 3	4	$\mathbf{D}_6 \times 2$	$Z_2 = \Phi$
8	1 + 4	3	$\mathbf{D}_{16}$	
9	1 + 2	2	$\mathbf{D}_{12} \times 2$	Z = N

p	**H**	**#**
2	$\mathbf{D}_{16}$	3
3	3	1

Max subgps

H	**#**
24	1
$\mathbf{D}_{24}$	2
$\mathbf{D}_{16}$	3

48.43 = $M_{24,2}^{(11)}$ = ⟨A^{24}, B^2, $BA = A^{11}B$⟩

Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6	Γ_7	Γ_8	Γ_9	Γ_{10}	Γ_{11}	Γ_{12}	Γ_{13}	Γ_{14}	Γ_{15}
1	A^{12}	A^4 A^{20}	A^8 A^{16}	A^6 A^{18}	A^2 A^{22}	A^{10} A^{14}	A^3 A^9	A^{15} A^{21}	A A^{11}	A^5 A^7	A^{13} A^{23}	A^{17} A^{19}	$A^{2n}B$	$A^{2n+1}B$

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
#	**1**	**1**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**12**	**12**
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1
χ_3	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	1	−1
χ_4	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	1
χ_5	2	2	2	2	−2	−2	−2	0	0	0	0	0	0	0	0
χ_6	2	2	−1	−1	2	−1	−1	2	2	−1	−1	−1	−1	0	0
χ_7	2	2	−1	−1	2	−1	−1	−2	−2	1	1	1	1	0	0
χ_8	2	2	−1	−1	−2	1	1	0	0	$\sqrt{3}$	$-\sqrt{3}$	$\sqrt{3}$	$-\sqrt{3}$	0	0
χ_9	2	2	−1	−1	−2	1	1	0	0	$-\sqrt{3}$	$\sqrt{3}$	$-\sqrt{3}$	$\sqrt{3}$	0	0
χ_{10}	2	−2	−2	2	0	0	0	$\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	0	0
χ_{11}	2	−2	−2	2	0	0	0	$-\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	0	$\sqrt{2}$	0	0
χ_{12}	2	−2	1	1	0	$\sqrt{3}$	$-\sqrt{3}$	$\sqrt{2}$	$-\sqrt{2}$	$\frac{\sqrt{6}+\sqrt{2}}{2}$	$\frac{\sqrt{6}-\sqrt{2}}{2}$	$\frac{-\sqrt{6}-\sqrt{2}}{2}$	$\frac{-\sqrt{6}+\sqrt{2}}{2}$	0	0
χ_{13}	2	−2	1	−1	0	$-\sqrt{3}$	$\sqrt{3}$	$-\sqrt{2}$	$\sqrt{2}$	$\frac{\sqrt{6}-\sqrt{2}}{2}$	$\frac{\sqrt{6}+\sqrt{2}}{2}$	$\frac{-\sqrt{6}+\sqrt{2}}{2}$	$\frac{-\sqrt{6}-\sqrt{2}}{2}$	0	0
χ_{14}	2	−2	1	−1	0	$-\sqrt{3}$	$\sqrt{3}$	$\sqrt{2}$	$-\sqrt{2}$	$\frac{-\sqrt{6}+\sqrt{2}}{2}$	$\frac{-\sqrt{6}-\sqrt{2}}{2}$	$\frac{\sqrt{6}-\sqrt{2}}{2}$	$\frac{\sqrt{6}+\sqrt{2}}{2}$	0	0
χ_{15}	2	−2	1	−1	0	$\sqrt{3}$	$-\sqrt{3}$	$-\sqrt{2}$	$\sqrt{2}$	$\frac{-\sqrt{6}-\sqrt{2}}{2}$	$\frac{-\sqrt{6}+\sqrt{2}}{2}$	$\frac{\sqrt{6}+\sqrt{2}}{2}$	$\frac{\sqrt{6}-\sqrt{2}}{2}$	0	0
^	**1**	**2**	**6**	**3**	**4**	**12**	**12**	**8**	**8**	**24**	**24**	**24**	**24**	**2**	**2**

	Classes	**H**	**G/H**
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13	24	2
2	1 + 2 + 3 + 4 + 5 + 6 + 7 + 14	$\mathbf{D}_{12} \times 2$	2
3	1 + 2 + 3 + 4 + 5 + 6 + 7 + 15	$\mathbf{D}_{12} \times 2$	2
4	1 + 2 + 3 + 4 + 5 + 6 + 7	12	2^2
5	1 + 2 + 5 + 8 + 9	8	$\mathbf{D}_6$
6	1 + 2 + 3 + 4	6	$\mathbf{D}_8$
7	1 + 2 + 5	4	$\mathbf{D}_6 \times 2$
8	1 + 4	3	$\mathbf{M}_{8,2}^{(3)}$
9	1 + 2	2	$\mathbf{D}_{12} \times 2$

G′	
$Z_2 = \Phi$	
Z = N	

p	**H**	**#**
2	$\mathbf{M}_{8,2}^{(3)}$	3
3	3	1

Max subgps

H	**#**
24	1
$\mathbf{D}_{24}$	1
$\mathbf{Q}_{24}$	1
$\mathbf{M}_{8,2}^{(3)}$	3

48.44 = ⟨A^{24}, $B^2 = A^{12}$, $BA = A^{-1}B$⟩

Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6	Γ_7	Γ_8	Γ_9	Γ_{10}	Γ_{11}	Γ_{12}	Γ_{13}	Γ_{14}	Γ_{15}
1	A^{12}	A^6 A^{18}	A^8 A^{16}	A^4 A^{20}	A^3 A^{21}	A^9 A^{15}	A^2 A^{22}	A^{10} A^{14}	A A^{23}	A^5 A^{19}	A^7 A^{17}	A^{11} A^{13}	$A^{2n}B$	$A^{2n+1}B$

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
#	1	1	2	2	2	2	2	2	2	2	2	2	2	12	12
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1
χ_3	1	1	1	1	1	−1	−1	1	1	−1	−1	−1	−1	1	−1
χ_4	1	1	1	1	1	−1	−1	1	1	−1	−1	−1	−1	−1	1
χ_5	2	2	2	−1	−1	2	2	−1	1	−1	−1	−1	−1	0	0
χ_6	2	2	2	−1	−1	−2	−2	−1	1	1	1	1	1	0	0
χ_7	2	2	−2	2	2	0	0	−2	−2	0	0	0	0	0	0
χ_8	2	−2	0	2	−2	$\sqrt{2}$	$-\sqrt{2}$	0	1	$-\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$	0	0
χ_9	2	−2	0	2	−1	$-\sqrt{2}$	$\sqrt{2}$	0	0	$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	0	0
χ_{10}	2	2	−2	−1	−1	0	0	1	1	$\sqrt{3}$	$-\sqrt{3}$	$-\sqrt{3}$	$\sqrt{3}$	0	0
χ_{11}	2	2	−2	−1	−1	0	0	1	1	$-\sqrt{3}$	$\sqrt{3}$	$\sqrt{3}$	$-\sqrt{3}$	0	0
χ_{12}	2	−2	0	−1	1	$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{3}$	$-\sqrt{3}$	$\frac{\sqrt{6}+\sqrt{2}}{2}$	$\frac{\sqrt{6}-\sqrt{2}}{2}$	$\frac{-\sqrt{6}+\sqrt{2}}{2}$	$\frac{-\sqrt{6}-\sqrt{2}}{2}$	0	0
χ_{13}	2	−2	0	−1	1	$-\sqrt{2}$	$\sqrt{2}$	$-\sqrt{3}$	$\sqrt{3}$	$\frac{\sqrt{6}-\sqrt{2}}{2}$	$\frac{\sqrt{6}+\sqrt{2}}{2}$	$\frac{-\sqrt{6}-\sqrt{2}}{2}$	$\frac{-\sqrt{6}+\sqrt{2}}{2}$	0	0
χ_{14}	2	−2	0	−1	1	$\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{3}$	$\sqrt{3}$	$\frac{-\sqrt{6}+\sqrt{2}}{2}$	$\frac{-\sqrt{6}-\sqrt{2}}{2}$	$\frac{\sqrt{6}+\sqrt{2}}{2}$	$\frac{\sqrt{6}-\sqrt{2}}{2}$	0	0
χ_{15}	2	−2	0	−1	1	$-\sqrt{2}$	$\sqrt{2}$	$\sqrt{3}$	$-\sqrt{3}$	$\frac{-\sqrt{6}-\sqrt{2}}{2}$	$\frac{-\sqrt{6}+\sqrt{2}}{2}$	$\frac{\sqrt{6}-\sqrt{2}}{2}$	$\frac{\sqrt{6}+\sqrt{2}}{2}$	0	0
^	**1**	**2**	**4**	**3**	**6**	**8**	**8**	**12**	**12**	**24**	**24**	**24**	**24**	**4**	**4**

	Classes	**H**	**G/H**
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13	24	2
2	1 + 2 + 3 + 4 + 5 + 8 + 9 + 14	**Q**$_{24}$	2
3	1 + 2 + 3 + 4 + 5 + 8 + 9 + 15	**Q**$_{24}$	2
4	1 + 2 + 3 + 4 + 5 + 8 + 9	12	2^2
5	1 + 2 + 3 + 6 + 7	8	**D**$_6$
6	1 + 2 + 4 + 5	6	**D**$_8$
7	1 + 2 + 3	4	**D**$_6$ × 2
8	1 + 4	3	**Q**$_{16}$
9	1 + 2	2	**D**$_{12}$ × 2

G′

$Z_2 = \Phi = N$

Z

p	**H**	**#**
2	**Q**$_{16}$	3
3	3	1

Max subgps

H	**#**
24	1
Q$_{24}$	2
8 × 2	3

48.45 = ⟨A^4, B^3, $C^2 = A^3$, D^2, $CB = B^{-1}C$, $DA = A^{-1}D$, $DC = ACD$⟩

Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6	Γ_7	Γ_8	Γ_9	Γ_{10}	Γ_{11}	Γ_{12}
1	A^2	B B^2	A^2B A^2B^2	A A^3	AB AB^2 A^3B A^3B^2	A^nD	A^nBD	A^nB^2D	B^nC AB^nC	A^2B^nC A^3B^nC	A^mB^n CD

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
#	1	1	2	2	2	4	4	4	4	6	6	12
χ_1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	1	–1	–1	–1
χ_3	1	1	1	1	1	1	–1	–1	–1	–1	–1	1
χ_4	1	1	1	1	1	1	–1	–1	–1	1	1	–1
χ_5	2	2	–1	–1	2	–1	2	–1	–1	0	0	0
χ_6	2	2	–1	–1	2	–1	–2	1	1	0	0	0
χ_7	2	2	2	2	–2	–2	0	0	0	0	0	0
χ_8	2	2	–1	–1	–2	1	0	$\sqrt{3}$ i	$-\sqrt{3}$ i	0	0	0
χ_9	2	2	–1	–1	–2	1	0	$-\sqrt{3}$ i	$\sqrt{3}$ i	0	0	0
χ_{10}	2	–2	2	–2	0	0	0	0	0	$\sqrt{2}$	$-\sqrt{2}$	0
χ_{11}	2	–2	2	–2	0	0	0	0	0	$-\sqrt{2}$	$\sqrt{2}$	0
χ_{12}	4	–4	–2	2	0	0	0	0	0	0	0	0
^	**1**	**2**	**3**	**6**	**4**	**12**	**2**	**6**	**6**	**8**	**8**	**2**

	Classes	**H**	**G/H**
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9	$\mathbf{D}_8 \times 3$	2
2	1 + 2 + 3 + 4 + 5 + 6 + 12	$\mathbf{D}_{12} \times 2$	2
3	1 + 2 + 3 + 4 + 5 + 6 + 10 + 11	$\mathbf{D}_{3,8}$	2
4	1 + 2 + 3 + 4 + 5 + 6	12	2^2
5	1 + 2 + 5 + 7	$\mathbf{D}_8$	$\mathbf{D}_6$
6	1 + 2 + 3 + 4	6	$\mathbf{D}_8$
7	1 + 2 + 5	4	$\mathbf{D}_6 \times 2$
8	1 + 3	3	$\mathbf{D}_{16}$
9	1 + 2	2	24.11

G′

$Z_2 = \Phi$

$Z = \mathfrak{A}$

p	**H**	**#**
2	$\mathbf{D}_{16}$	3
3	3	1

Max subgps

H	**#**
$\mathbf{D}_8 \times 3$	1
$\mathbf{D}_{3,8}$	1
$\mathbf{D}_{24}$	1
$\mathbf{D}_{16}$	3

48.46 = ⟨A^3, B^8, C^2, BA = $A^{-1}B$, CB = B^3C⟩

Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6	Γ_7	Γ_8	Γ_9	Γ_{10}	Γ_{11}	Γ_{12}
1	A^2	B B^2	A^2B A^2B^2	A A^3	AB AB^2 A^3B A^3B^2	D AD A^2D A^3D	BD ABD A^2BD A^3BD	B^2D AB^2D A^2B^2D A^3B^2D	C AC BC ABC B^2C AB^2C	A^2C A^3C A^2BC A^3BC A^2B^2C A^3B^2C	A^nCD A^nBCD A^nB^2CD

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
#	**1**	**1**	**2**	**2**	**2**	**4**	**4**	**4**	**4**	**6**	**6**	**12**
χ_1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	1	–1	–1	–1
χ_3	1	1	1	1	1	1	–1	–1	–1	–1	–1	1
χ_4	1	1	1	1	1	1	–1	–1	–1	1	1	–1
χ_5	2	2	–1	–1	2	–1	2	–1	–1	0	0	0
χ_6	2	2	–1	–1	2	–1	–2	1	1	0	0	0
χ_7	2	2	2	2	–2	–2	0	0	0	0	0	0
χ_8	2	2	–1	–1	–2	1	0	$\sqrt{3}$ i	$-\sqrt{3}$ i	0	0	0
χ_9	2	2	–1	–1	–2	1	0	$-\sqrt{3}$ i	$\sqrt{3}$ i	0	0	0
χ_{10}	2	–2	2	–2	0	0	0	0	0	$\sqrt{2}$	$-\sqrt{2}$	0
χ_{11}	2	–2	2	–2	0	0	0	0	0	$-\sqrt{2}$	$\sqrt{2}$	0
χ_{12}	4	–4	–2	2	0	0	0	0	0	0	0	0
order	1	2	3	6	4	12	2	6	6	8	8	2

	Classes	**H**	**G/H**	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9	$\mathbf{D}_8 \times 3$	2	
2	1 + 2 + 3 + 4 + 5 + 6 + 12	$\mathbf{D}_{12} \times 2$	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 10 + 11	$\mathbf{D}_{3,8}$	2	
4	1 + 2 + 3 + 4 + 5 + 6	12	2^2	G′
5	1 + 2 + 5 + 7	$\mathbf{D}_8$	$\mathbf{D}_6$	
6	1 + 2 + 3 + 4	6	$\mathbf{D}_8$	
7	1 + 2 + 5	4	$\mathbf{D}_6 \times 2$	$Z_2 = \Phi$
8	1 + 3	3	$\mathbf{D}_{16}$	
9	1 + 2	2	24.11	Z = N

p	**H**	**#**
2	$\mathbf{M}_{8,2}^{(3)}$	3
3	3	1

Max subgps

H	**#**
$\mathbf{D}_8 \times 3$	1
$\mathbf{D}_{3,8}$	1
$\mathbf{Q}_{24}$	1
$\mathbf{M}_{8,2}^{(3)}$	3

48.47 = ⟨A^3, B^8, C^2, $BA = A^{-1}B$, $CB = B^3C$⟩

Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6	Γ_7	Γ_8	Γ_9	Γ_{10}	Γ_{11}	Γ_{12}
1	B^4	A A^2	AB^4 A^2B^4	B^2 B^6	AB^2 A^2B^2 AB^6 A^2B^6	C B^2C B^4C B^6C	AC A^2B^2C AB^4C A^2B^6C	A^2C B^2C A^2B^4C B^6C	A^nB A^nB^3	A^nB^5 A^nB^7	A^nBC A^nB^3C A^nB^5C A^nB^7C

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
#	**1**	**1**	**2**	**2**	**2**	**4**	**4**	**4**	**4**	**6**	**6**	**12**
χ_1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	1	–1	–1	–1
χ_3	1	1	1	1	1	1	–1	–1	–1	–1	–1	1
χ_4	1	1	1	1	1	1	–1	–1	–1	1	1	–1
χ_5	2	2	–1	–1	2	–1	2	–1	–1	0	0	0
χ_6	2	2	–1	–1	2	–1	–2	1	1	0	0	0
χ_7	2	2	2	2	–2	–2	0	0	0	0	0	0
χ_8	2	2	–1	–1	–2	1	0	$\sqrt{3}\,i$	$-\sqrt{3}\,i$	0	0	0
χ_9	2	2	–1	–1	–2	1	0	$-\sqrt{3}\,i$	$\sqrt{3}\,i$	0	0	0
χ_{10}	2	–2	2	–2	0	0	0	0	0	$\sqrt{2}\,i$	$-\sqrt{2}\,i$	0
χ_{11}	2	–2	2	–2	0	0	0	0	0	$-\sqrt{2}\,i$	$\sqrt{2}\,i$	0
χ_{12}	4	–4	–2	2	0	0	0	0	0	0	0	0
order	1	2	3	6	4	12	2	6	6	8	8	4

	Classes	**H**	**G/H**
1	1 2 3 4 5 6 7 8 9	$\mathbf{D}_8 \times 3$	2
2	1 2 3 4 5 6 12	$\mathbf{Q}_{24}$	2
3	1 2 3 4 5 6 10 11	$\mathbf{D}_{3,8}$	2
4	1 2 3 4 5 6	12	2^2
5	1 2 5 7	$\mathbf{D}_8$	$\mathbf{D}_6$
6	1 2 3 4	6	$\mathbf{D}_8$
7	1 2 5	4	$\mathbf{D}_6 \times 2$
8	1 3	3	$\mathbf{D}_{16}$
9	1 2	2	24.11

G′

$Z_2 = \Phi$

Z = N

p	**H**	**#**
2	$\mathbf{M}_{8,2}^{(3)}$	3
3	3	1

Max subgps

H	**#**
$\mathbf{Q}_8 \times 3$	1
$\mathbf{D}_{3,8}$	1
$\mathbf{D}_{24}$	1
$\mathbf{M}_{8,2}^{(3)}$	3

48.48 = ⟨A^3, B^8, $C^2 = B^4$, $BA = A^{-1}B$, $CB = B^{-1}C$⟩

CC	1	2	3	4	5	6	7	8	9	10	11	12
elts	1	B^4	B^2 B^6	A A^2	AB^4 A^2B^4	AB^2 A^2B^2 AB^6 A^2B^6	A^nB A^nB^7	A^nB^3 A^nB^5	$A^nB^{2n+1}C$	C B^2C B^4C B^6C	AC AB^4C A^2B^2C A^2B^6C	AB^2C AB^6C A^2C A^2B^4C

C	1	2	3	4	5	6	7	8	9	10	11	12	
#	1	1	2	2	2	4	6	6	12	4	4	4	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	–1	–1	–1	1	1	1	1
χ_3	1	1	1	1	1	1	–1	–1	1	–1	–1	–1	2
χ_4	1	1	1	1	1	1	1	1	–1	–1	–1	–1	3
χ_5	2	2	2	–1	–1	–1	0	0	0	2	–1	–1	4
χ_6	2	2	2	–1	–1	–1	0	0	0	–2	1	1	5
χ_7	2	2	–2	2	2	–2	0	0	0	0	0	0	6
χ_8	2	2	–2	–1	–1	1	0	0	0	0	$\sqrt{3}i$	$-\sqrt{3}i$	7
χ_9	2	2	–2	–1	–1	1	0	0	0	0	$-\sqrt{3}i$	$\sqrt{3}i$	7
χ_{10}	2	–2	0	2	–2	0	$\sqrt{2}$	$-\sqrt{2}$	0	0	0	0	8
χ_{11}	2	–2	0	2	–2	0	$-\sqrt{2}$	$\sqrt{2}$	0	0	0	0	8
χ_{12}	4	–4	0	–2	2	0	0	0	0	0	0	0	0
^	1	2	4	3	6	12	8	8	2	4	12	12	

	Classes	H	G/H		
1	1 + 2 + 3 + 4 + 5 + 6 + 10 + 11 + 12	**$Q_8 \times 3$**	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 9	**$D_{12} \times 2$**	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	**$D_{3,8}$**	**2**	3	
4	1 + 2 + 3 + 10	**Q_8**	**D_6**	4	
5	1 + 2 + 3	**4**	**$D_6 \times 2$**	5	**Φ = ℜ**
6	1 + 2 + 4 + 5	**6**	**D_8**	6	
7	1 + 2	**2**	**24.11**	7	**Z**
8	1 + 4	**3**	**Q_{16}**	8	
9	1 + 2 + 3 + 4 + 5 + 6	**12**	**2^2**	1∩2	**G′**

Sylow subgroups: [Q_{16}] × 3, [3] **Maximal subgroups:** [$Q_8 \times 3$], [$D_{3,8}$], [Q_{24}]

48.49 = $GL_{2,3}$ = ⟨A^4, $B^2 = A^2$, C^3, D^2, $BA = A^{-1}B$, $CA = A^3BC$, $CB = A^3C$, $DA = BD$, $DB = AD$, $DC = C^{-1}D$⟩

CC	1	2	3	4	5	6	7	8
elts	1	A^2	A A^3 B AB A^2B A^3B	AC^nD BC^nD	A^3C^nD A^2BC^nD	C A^2BC A^3BC AC C^2 A^3C^2 ABC^2 BC^2	A^2C A^3C BC ABC AC^2 A^2C^2 A^2BC^2 A^3BC^2	C^nD A^2C^nD ABC^nD A^3BC^nD

C	1	2	3	4	5	6	7	8	
#	1	1	6	6	6	8	8	12	ℵ
χ_1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	–1	–1	1	1	–1	1
χ_3	2	2	2	0	0	–1	–1	0	2
χ_4	2	–2	0	$\sqrt{2}\,i$	$-\sqrt{2}\,i$	–1	1	0	0
χ_5	2	2	0	$-\sqrt{2}\,i$	$\sqrt{2}\,i$	–1	1	0	0
χ_6	3	3	–1	–1	–1	0	0	1	3
χ_7	3	3	–1	1	1	0	0	–1	3
χ_8	4	–4	0	0	0	1	–1	0	0
^	1	2	4	8	8	3	6	2	

	Classes	H	G/H		
1	1 + 2 + 3 + 6 + 7	$SL_{2,3}$	2	1	G′
2	1 + 2 + 3	Q_8	D_6	2	
3	1 + 2	2	S_4	3	Z = Φ = 𝔄

Sylow subgroups: [$M_{8,2}^{(3)}$] × 3, [3] × 4
Maximal subgroups: [$SL_{2,3}$], [$M_{8,2}^{(3)}$] × 3, [$D_6 \times 2$] × 4

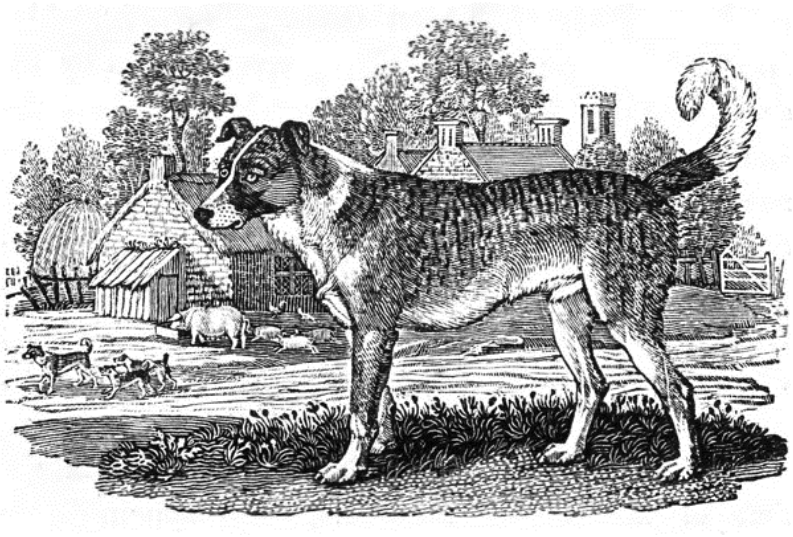
48.50 = ⟨A^4, $B^2 = A^2$, C^3, $D^2 = A^2$, $BA = A^{-1}B$, $CA = A^3BC$, $CB = A^3C$, $DA = BD$, $DB = AD$, $DC = C^{-1}D$⟩

CC	1	2	3	4	5	6	7	8
elts	1	A^2	A A^3 B AB A^2B A^3B	AC^nD BC^nD	A^3C^nD A^2BC^nD	C A^2BC A^3BC AC C^2 A^3C^2 ABC^2 BC^2	A^2C A^3C BC ABC AC^2 A^2C^2 A^2BC^2 A^3BC^2	C^nD A^2C^nD ABC^nD A^3BC^nD

C	1	2	3	4	5	6	7	8	
#	1	1	6	6	6	8	8	12	$\mathcal{K}$
χ_1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	–1	–1	1	1	–1	1
χ_3	2	2	2	0	0	–1	–1	0	2
χ_4	2	–2	0	$\sqrt{2}\,i$	$-\sqrt{2}\,i$	–1	1	0	0
χ_5	2	–2	0	$-\sqrt{2}\,i$	$\sqrt{2}\,i$	–1	1	0	0
χ_6	3	3	–1	–1	–1	0	0	1	3
χ_7	3	3	–1	1	1	0	0	–1	3
χ_8	4	–4	0	0	0	1	–1	0	0
^	1	2	4	8	8	3	6	4	

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 6 + 7	$SL_{2,3}$	2	1	G′
2	1 + 2 + 3	Q_8	D_6	2	
3	1 + 2	2	S_4	3	Z = Φ = $\mathfrak{A}$

Sylow subgroups: [Q_{16}] × 3, [3] × 4 **Maximal subgroups:** [$SL_{2,3}$], [Q_{16}] × 3, [$D_{3,4}$] × 4



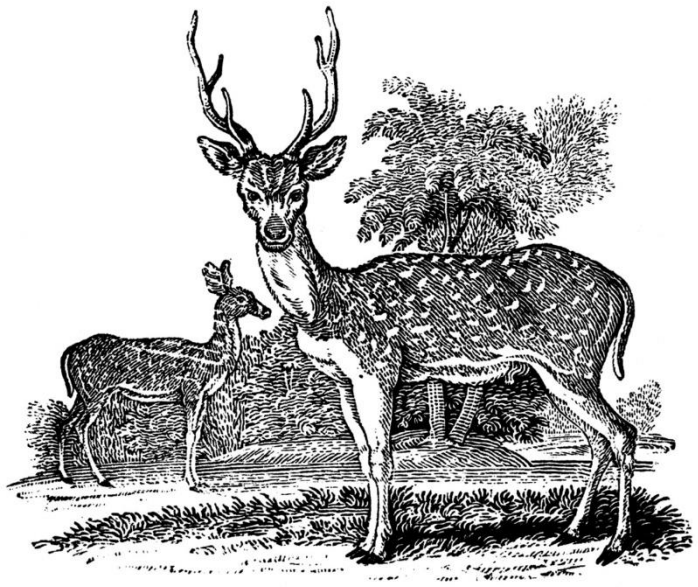
48.51 = ⟨A^2, B^2, C^2, D^2, E^3, EA = ABE, EB = AE, EC = CDE, ED = CE⟩

CC	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
elts	1	A B AB	C D CD	BC ABD ACD	AD ABC BCD	AC BD ABCD	$A^mB^nC^rD^sE$	$A^mB^nC^rD^sE^2$

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
#	1	3	3	3	3	3	16	16	$\mathcal{K}$
χ_1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	ω	ω^2	1
χ_3	1	1	1	1	1	1	ω^2	ω	1
χ_4	3	3	–1	–1	–1	–1	0	0	2
χ_5	3	–1	3	–1	–1	–1	0	0	3
χ_6	3	3	–1	3	–1	–1	0	0	4
χ_7	3	3	–1	–1	3	–1	0	0	5
χ_8	3	–1	–1	–1	–1	3	0	0	6
^	**1**	**2**	**2**	**2**	**2**	**2**	**3**	**3**	

	Classes	**H**	**G/H**	∩	
1	1 + 2 + 3 + 4 + 5 + 6	$\mathbf{2^4}$	**3**	1	G′
2	1 + 2	$\mathbf{2^2}$	$\mathbf{A_4}$	2	
3	1 + 3	$\mathbf{2^2}$	$\mathbf{A_4}$	3	
4	1 + 2 + 4	$\mathbf{2^2}$	$\mathbf{A_4}$	4	
5	1 + 2 + 5	$\mathbf{2^2}$	$\mathbf{A_4}$	5	
6	1 + 6	$\mathbf{2^2}$	$\mathbf{A_4}$	6	

Sylow subgroups: [$\mathbf{2^4}$], [**3**] × 16 **Maximal subgroups:** [$\mathbf{2^4}$], [$\mathbf{A_4}$] × 4



48.52 = ⟨A^4, B^4, C^3, $CA = A^3B^3C$, $CB = AC$⟩

CC	1	2	3	4	5	6	7	8
elts	1	A^2 B^2 A^2B^2	A B A^3B^3	A^3 B^3 AB^3	AB^2 A^2B^3 AB^3	A^2B A^3B A^3B^2	A^mB^nC	$A^mB^nC^2$

C	1	2	3	4	5	6	7	8	
#	1	3	3	3	3	3	16	16	$\mathcal{K}$
χ_1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	ω	ω^2	1
χ_3	1	1	1	1	1	1	ω^2	ω	1
χ_4	3	3	–1	–1	–1	–1	0	0	2
χ_5	3	–1	1	1	–1 + 2i	–1 –2i	0	0	0
χ_6	3	–1	1	1	–1 –2i	–1 + 2i	0	0	0
χ_7	3	–1	–1 + 2i	–1 –2i	1	1	0	0	0
χ_8	3	–1	–1 –2i	–1 + 2i	1	1	0	0	0
^	1	2	4	4	4	4	3	3	

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6	$\mathbf{4^2}$	**3**	1	G′
2	1 + 2	$\mathbf{2^2}$	$\mathbf{A_4}$	2	Φ

Sylow sugroups: [$\mathbf{4^2}$], [**3**] × 16 **Maximal subgroups:** [$\mathbf{4^2}$], [$\mathbf{A_4}$] × 4